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# MEASURING SYSTEMIC RISK IN FINANCIAL INSTITUTIONS: A FACTOR-COPULA FRAMEWORK

Master's Thesis submitted to

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## **Abstract**

This work proposes a factor copula model to quantify systemic risk in financial institutions. This framework connects to recently trending research on factor copula modeling and systemic risk measurement. The underlying data are equity returns of the 28 systemically important financial institutions and a common factor which is a portfolio being weighted by these SIFIs. Considering a one-factor copula model with distributional assumptions that enable asymmetric and tail dependence, this framework provides great fit to the underlying financial data. The estimation of the copula density expression is accomplished by the Gauss-Legendre quadratures, a numerical integration and optimization procedure to solve expressions without analytical solutions. Dependence measures and tail dependence coefficients are obtained based on the factor copula framework and on a nonparametric approach. Both tail dependence measures, though estimated by a parametric and a nonparametric approach, yield results implying a higher tail dependence among the SIFIs in 2015. Then, this work introduces recognized risk measures which become compared in their appropriateness in measuring systemic risk. The focus of the chosen risk measures is to estimate the risk exposure of the financial institutions to the financial system. Hence, the vulnerability of the individual banks is assessed and results indicate again increasing exposure in 2015.

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## List of Abbreviations

<b>cdf</b>	cumulative distribution function
<b>CDS</b>	Credit Default Swap
<b>CoVaR</b>	Conditional Value-at-Risk
<b>ES</b>	Expected Shortfall
<b>ExCoVaR</b>	Exposure $\Delta$ Conditional Value-at-Risk
<b>GARCH</b>	Generalized Autoregressive conditional heteroscedasticity
<b>MES</b>	Marginal Expected Shortfall
<b>MLE</b>	Maximum Likelihood Estimation
<b>pdf</b>	probability density function
<b>PIT</b>	Probability Integral Transform
<b>SIFI</b>	systemically important financial institutions
<b>VaR</b>	Value-at-Risk

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# 1 Introduction

What severe consequences does distress in the financial system have on the systemically important financial institutions? Concerning the volatile financial markets over the last decade, in particular the financial crisis and the European sovereign debt crisis, systemic risk has emerged as trending topic among the financial sector and regulators. With regards to the Lehman Brothers default and the subsequent turmoil in the banking sector, the interconnectedness of the financial institutions attracted the attention of the public. From then on, large and interconnected financial institutions were seen as systemic risk drivers and the focus of regulators shifted to the arising topics of *too-big-to-fail* and *too-interconnected-to-fail*. While the attention of regulators and risk managers moved on to an adequate measuring of systemic risk, dependence and risk measures have been studied in higher frequency.

Risk measures such as Value-at-Risk and Expected Shortfall became criticized for being inappropriate approaches on the quantification of systemic risk born by financial institutions. As a result, Adrian and Brunnermeier (2008) and Acharya et al. (2017) introduce risk measures conditional on distressed events. That are the Conditional Value-at-Risk (CoVaR) and the Marginal Expected Shortfall (MES) which hold as more reliable measures of systemic risk. Further research by Mainik and Schaanning (2014) and Girardi and Ergün (2013) extend these measures to address different definitions of systemic risk and to improve their robustness.

Whereas the most common dependence measures concentrate on linear and symmetric dependence and thus yield inadequate estimations during extreme events. Derived from variance-covariance estimation a large number of models belong to the correlation-based approaches which risk quantifications might be an appropriate way under normal market conditions but lack under non-normal market behavior. Since such approaches often assume normality of the risk factors, they misspecify risk by underestimating the dependence between risk factors being in a tail event. In Schmidt and Stadtmüller (2006) tail dependence measures have been introduced to capture more adequately pairwise dependencies in the tails between financial institutions.

While pairwise dependence measures might not hold as system-wide risk measures that incorporate simultaneous dependencies within high-dimensional applications, the copula function offers a flexible and powerful alternative to traditional dependence coefficients. Joint distribution functions can be decomposed into a copula, containing the entire information set about the dependence structure, and marginal distributions. This property enables flexible constructions for joint distributions by imposing different underlying distributional assumptions. Furthermore, copulas provide the possibility of multi-stage

estimation which essentially enhances the computation due to dimension reduction.

In particular when dealing with applications on financial data, the normality and symmetry assumptions fail to hold. On the basis of financial applications, the normality assumption would rule out the dependence of joint tail events and the symmetry assumption would not capture the potentially asymmetric dependence which assigns a higher likelihood to joint downturns than to upturns. While many approaches are fitting copulas to two dimensional models, appropriate copulas which are able to model high-dimensional problems have been revealed by recent research.

Factor models introduced by Krupskii and Joe (2013) became a powerful tool for dependence modeling on systemic risk applications. In past literature on factor copula models though (He and Gong, 2009), Gaussian frameworks have been employed frequently due to their attractive feature regarding the estimation of the cumulative distribution function. While heavy tailed distributions are more suitable to capture the characteristics of financial data, they are not stable under convolution and even worse, the distribution does not offer a closed-form solution. Hence, the estimation and corresponding computation require numerical integration and optimization procedures which distinctly raise the computational efforts and thus create incentives for dimension reduction.

This work addresses this issue by applying a copula model on a factor structure and imposing heavy tailed distributions. Firstly, the distributional assumption allows for tail dependence within the factor model which secondly, is a powerful tool to reduce dimensions being particularly useful for high-dimensional applications. Moreover, factor models only reduce the dimensions for the copula estimation while the marginal distributions are estimated separately within the multi-stage estimation and thus maintain unaffected by the dimension reduction. Oh and Patton (2017a) provide research on dependence modeling under different heavy tailed distributional assumptions. Extensive research on dependence modeling using copulas has been provided by Joe (2014) who applied archimedean copulas on the factor model. Chen and Nasekin (2017) analyzed systemic risk within a network-based factor copula which conditions the network model on a central node which is the most interconnected financial institution. Vrms (2009), Choroś-Tomczyka et al. (2012) and Chen and Nasekin (2017) are based on double  $t$  frameworks enabling tail dependence.

However, this work proposes a skewed distribution to the systemic factor and hence induces asymmetric tail dependence. The structure of the factor model makes the factor copula being solely determined by the distributions for the common factors and the idiosyncratic components. In this work, the underlying financial data are equity returns of the 28 systemically important financial institutions from pre-crisis period 2007 until

2015. While the return series of these institutions are used for the idiosyncratic shock, a portfolio weighted by the respective individual sizes represents the common factor. Hence, imposing a heavy tailed and skewed distribution to the systemic factor and a symmetric heavy tailed distributions to the idiosyncratic components results to the desired tail dependence and asymmetry which provides a better fit to the financial data.

Once the one-factor structure is introduced, the conditional copula expression is derived after applying an uniform transformation method. While this work considers a Maximum likelihood estimation for the copula density, Oh and Patton (2013) propose the simulated method of moments estimation method to factor copula models which do not provide closed-form solutions. Since many heavy tailed copula frameworks are not solvable analytically, the likelihood expression must be obtained by the means of numerical integration methods. Thereby, quadrature methods are presumed to yield adequate results. Whereas Vršina (2009) applied Gauss-Hermite quadratures, which sometimes lack of bounded approximation errors, Gauss-Legendre quadratures address this issue and are employed by Oh and Patton (2017b) and Chen and Nasekin (2017). Then, the resulting simulated returns are the underlying series for the estimations of the risk measures. Thereby, after obtaining the popular measures of VaR and ES, this approach will focus on the vulnerability of each financial institution to the financial system and hence takes the Exposure CoVaR approach by Adrian and Brunnermeier (2008) and the Marginal Expected Shortfall by Acharya et al. (2017) into account. This setting then focuses on the event when a system-wide shock occurred and investigates the subsequent impact on all financial institutions based on a simultaneous analysis. Hence, this work follow up to the analysis of existing research which studied the systemic risk born by institution-specific extreme tail events and the resulting effect on the financial system. This has been studied by, among others, Girardi and Ergün (2013), Mainik and Schaanning (2014), Cao (2013) in a multivariate framework and Chen and Nasekin (2017) in a network-based factor copula model .

While this work tries to quantify the systemic risk in financial institutions, it examines dependences structures within the financial system and tail dependencies between the banks themselves at first and continues to assess the appropriateness of the proposed risk measures. The dependence within the financial system increases during more volatile periods as it is implied by the distributional assumptions. Though, results for the most recent year 2015 show higher dependence again, indicating an increasing interconnectedness in the financial system after periods of decreasing dependence structures. In order to the factor copula parameters and the tail dependence coefficients, clustered dependencies emerge according to the bank's region. Hence, the results show relatively low dependence



from the Asian financial sector on the overall and in particular Western banking sector. In line with the results on dependencies, the risk measures also yield comprehensible findings marking crisis periods quite well while they also identify higher individual systemic risk exposure of all institutions to the financial system in 2015. Thereby, the conditional risk measures obtain some results that are not yielded by the simple risk measures of VaR and ES which can be traced back to the different nature of the risk measurement approaches, such as conditioning on a stressed system. Anyway, the broad consistency of the risk estimates' and the dependence measures' results does not only lead back to the common model-approach on which the estimates are based on since even the nonparametric tail dependence supports the other estimates and suggests a higher interconnectedness within the financial system in 2015.

The structure of this work is constructed as follows. In section 2 basic theory on copulas, dependence measures and factor model are presented to continue with the construction of the factor-copula framework. This section on the methodological background is followed by the estimation part 3 which applies the Maximum Likelihood estimation method to obtain a solution to the copula density estimation. While this framework assumes heavy tail distributions on the one-factor copula increments, the Maximum Likelihood estimation requires numerical integration and optimization methods to achieve a solution. Hence, the subsequent section 3.2 deals with the methodology, computational implementation and existing research applications on numerical solution methods. Then, the work turns to the section 4 that discusses the adequacy of prevalent risk measures for estimating systemic risk in financial institutions. Finally, the results section 5 presents useful estimates to dependencies within the financial system and compares the risk measure estimates in the course of their appropriateness in quantifying systemic risk and according to their vulnerability to a shock of the financial system.

## 2 Methodology

This chapter is constructed as follows. First, basic concepts on copula and dependence measures are introduced. The main part of this chapter deals with the factor copula model, the implementation of the distributional assumptions and the derivation of the copula density function.

### 2.1 Theory on Copula

Consider for the  $d$ -variate stochastic process  $\{Y\}_{t=1}^n$  with  $Y_t = (y_{1,t}, \dots, y_{d,t})$  the joint distribution  $F(y_{1,t}, \dots, y_{d,t})$  and the corresponding marginal distribution  $F_i$  and density function  $f_i$  of each variable  $y_{i,t}$ .

By Sklar's Theorem (Sklar, 1959) the joint distribution can be expressed by the  $d$  marginal distributions which are "coupled" together by a copula function. A  $d$ -dimensional copula is a function  $C$  from  $[0, 1]^d \rightarrow [0, 1]$  that contains all information about the dependence structure. Hence,

$$F(Y_t) = C(F_1(y_{1,t}), \dots, F_d(y_{d,t})) \quad (1)$$

Thereby, the copula can be seen as a multivariate distribution function linking marginal distributions  $F_1, \dots, F_d$  to the joint distribution  $F$ . Sklar's Theorem provides following practical properties. Firstly, using the given univariate distributions  $F_1, \dots, F_d$  and a copula  $C$ , the theorem allows for the flexible construction of joint distributions  $F$ . Secondly, Sklar's Theorem enables multi-stage estimation which considerably lower the computational efforts when dealing with high-dimensional problems. The copula estimation consists of the estimation of the copula parameters  $\theta_C$  and the margins  $F_i$ ,  $i = 1, \dots, d$ . Depending on the practical application, the maximum likelihood estimation of the margins can be accomplished in a non-parametric model, which is in particular appropriate if the focus is only on the dependence structure, or in a parametric model. Latter one is more often applied in practical problems in which the entire distribution is of relevance. When applying the nonparametric approach, the maximum likelihood (ML) estimator should be proven to yield consistency and asymptotic normality. Within the parametric procedure, the ML estimation can be accomplished simultaneously for the parameter estimation of the margins and of the copula. This simultaneous (one-stage) estimation is more efficient than any multi-stage estimation procedure but brings an higher computational burden for high-dimensional applications (Genest et al., 1995). Thus, this framework applies the computationally attractive parametric two-stage estimation in which the parameters of margins are estimated in first stage and the copula parameters in the second stage.

Assuming an arbitrary continuous multivariate distribution which can be modeled parametrically, the probability integral transform (PIT) can be applied on equation (1) by imposing  $u_{i,t} = F_i(y_{i,t}; \theta_{m,i})$ , where  $u_{i,t} \sim Uniform(0,1)$  and  $\theta_{m,i}$  is the vector for all parameters of margin  $i$ . Then,  $\delta$  is the vector of all marginal parameters for  $i = 1, \dots, d$ . Since copulas are invariant under strictly increasing transformations, the PIT used on the random variables does not affect the copula functions.

$$C(u_1, \dots, u_d) = F(F_1^{-1}(u_1), \dots, F_d^{-1}(u_d)) \quad (2)$$

where  $F_i^{-1}$  is the generalized inverse of  $F_i$ . Hence, the copula can be described as a multivariate distribution with uniform margins and by considering the uniform transforms for equation (1), the copula density becomes:

$$c(u_{1,t}, \dots, u_{d,t}) = \frac{\partial^d C(u_1, \dots, u_d)}{\partial u_1 \dots \partial u_d} \quad (3)$$

Under the assumptions of a continuous multivariate distribution and a parametric model, one can also assume by Sklar's Theorem and by mixture families models the univariate marginal distribution and the copula to be continuous and parametric. Hence, equation (1) and (3) can be written as (Franke et al., 2004):

$$f(Y_t) = \prod_{j=1}^d f_j(y_{j,t}) \cdot c(u_{1,t}, \dots, u_{d,t}) \quad (4)$$

where  $f$  represents the joint density of  $Y_t$ . According to equation (4) which consists only of marginal densities  $f_i$  and the copula on the right hand side, it becomes obvious that the copula provides all information regarding the dependence structure.

## 2.2 Tail Dependence Properties

There are several approaches on the definition of pairwise dependence in random variables. The most common measures reflect linear and symmetric dependence or consider the entire distribution. In contrast, dependence can also be defined to be asymmetric, nonlinear or only consider the tail distribution. Thereby, dependence can be measured in an empirical way or model-implied (Chen and Nasekin, 2017).

When models with flexible dependence structures are employed, the linear correlation measure which is natively used under the multinormal assumption needs to become augmented by other dependence measures. The commonly employed linear correlation coefficient is not scale invariant and changes due to monotone increasing transformations.

That is the case because linear correlation is affected by the marginal distribution while in a copula framework appropriate dependence measures should only depend on the copula. Due to this property these measures provide information about the degree to which random variables gather around a monotone function (Joe, 1990).

That is why, an adequate dependence measure for a copula model is scale invariant (see Nelson 2006) and independent of strictly increasing transformations of the random variables such as the probability integral transformation which is described earlier in this framework. Following simulated rank dependence measures are common augmentations on the linear correlation since they are functions of the copula only (Patton et al., 2012). Therefore, consider the copula framework with two random variables  $y_{i,t}$  and  $y_{j,t}$ , their corresponding marginal distributions  $F_i$  and  $F_j$  and the copula  $C$ .

**Spearman's rank correlation** between random variables is measured by using the variables' concordance and discordance and can be considered as the linear correlation of the ranks of the simulated data. The population rank correlation  $\rho$  can be written as:

$$\begin{aligned}\rho &= Corr(u_{i,t}, u_{j,t}) = 12\mathbb{E}(u_{i,t}, u_{j,t}) - 3 \\ &= 12 \int_0^1 \int_0^1 C(u_i, u_j) du_i du_j - 3\end{aligned}$$

where  $\mathbb{E}(u) = 1/2$  and  $\mathbb{V}(u) = 1/12$  for any random variable  $u \sim Uniform(0, 1)$ . Then, the sample rank correlation  $\hat{\rho}$  can be estimated as:

$$\hat{\rho} = \frac{12}{n} \sum_{t=1}^n u_{i,t} u_{j,t} - 3$$

**Kendall's tau** between random variables is obtained by the difference in the probabilities of concordance and discordance. The Kendall's tau is calculated by:

$$\begin{aligned}\tau &= 4\mathbb{E}(C(u_{i,t}, u_{j,t})) - 1 \\ &= 4 \int_0^1 \int_0^1 C(u_i, u_j) dC(u_i, u_j) - 1\end{aligned}$$

As Spearman's and Kendall's rank correlation coefficients are defined in the interval  $[-1, 1]$  the direction of the dependence is given. These both measures may offer the most suitable alternatives to the linear correlation measure when dealing with dependence for non-elliptical distributions (Embrechts et al., 2001).

In contrast to the rank correlation coefficient above, the **Quantile dependence** focuses on the dependence in the joint tails of two random variables. It is the conditional prob-

ability of one variable being in the  $q$ -th quantile conditioned that the other variable is in the  $q$ -th quantile. On the one hand, while obtaining the lower and upper quantile dependence, information about the symmetry of the dependence structure are provided. On the other hand, this measure is defined in the interval  $[0,1]$  and thus, only shows the degree of dependence but tells nothing about the direction as the previous coefficients do (Patton et al., 2012).

**Tail dependence** is besides Quantile dependence another dependence measure of extreme events. This approach focuses on the upper-right and lower-left quadrant tail of a bivariate distribution of two random variables and therefore it measures the strength of dependence in the tails of a bivariate distribution. Since this analysis focuses on measuring risk in the financial markets under severely adverse market conditions the lower tail dependence coefficient is of higher relevance. In general, the tail dependence is defined by:

$$\tau_{ij}^L \equiv \lim_{q \rightarrow 0} \frac{P[X_i \leq F_i^{-1}(q), X_j \leq F_j^{-1}(q)]}{q} \quad (5)$$

$$\tau_{ij}^U \equiv \lim_{q \rightarrow 1} \frac{P[X_i > F_i^{-1}(q), X_j > F_j^{-1}(q)]}{1 - q} \quad (6)$$

The lower tail dependence gives the probability that both variables are below their  $q$  quantiles scaled by the probability of one of these variables being below its  $q$  quantile for the lower bound  $q \rightarrow 0$  (Oh and Patton, 2017a).

Hence, equation (5) and (6) are the limits of the quantile dependence and by embedding them into a copula framework it takes following form (De Luca and Riviello, 2012):

$$\tau_{ij}^L = \lim_{q \rightarrow 0} \frac{C(q, q)}{q} \quad (7)$$

$$\tau_{ij}^U = \lim_{q \rightarrow 1} \frac{1 - 2q - C(q, q)}{1 - q} \quad (8)$$

Whereas the majority of factor-copulas have no closed-form solution, the copula-implied tail dependence coefficients can be obtained in an analytical way utilizing results from extreme value theory. Based on the simple linear framework of a factor model the results for the copula-implied dependence are easy to calculate. Consider the one-factor copula model from equation (14) and under the assumptions that  $F_Z$  and  $F_\epsilon$  have regularly varying tails with a common tail index  $\alpha > 0$  and that the copula parameter of each SIFI bank  $\theta > 0$  holds, the tail dependence coefficients are given by equation (9) and (10).

$$\tau_{ij}^L = \frac{\min(\theta_i, \theta_j)^\alpha A_Z^L}{\min(\theta_i, \theta_j)^\alpha A_Z^L + A_\epsilon^L} \quad (9)$$

$$\tau_{ij}^U = \frac{\min(\theta_i, \theta_j)^\alpha A_Z^U}{\min(\theta_i, \theta_j)^\alpha A_Z^U + A_\varepsilon^U} \quad (10)$$

where  $A_Z^L$ ,  $A_Z^U$ ,  $A_\varepsilon^L$  and  $A_\varepsilon^U$  are constants which are given by the respective distributions. Since the implied tail dependencies are conditioned on a selected factor, they are also named conditional tail dependencies. In this case, the tail dependencies are conditioned on the systemic factor  $Z$ . Note, that the selections of the conditioning factor and the copula mainly determine the final tail dependencies (Chen and Nasekin, 2017).

According to the factor copula model from equation (14) lower and upper tail dependence coefficients are obtained if the sign of the copula parameters and the tail index of  $Z$  and  $\varepsilon$  are the same. The tail dependencies will be different and thus, enables to model different probabilities of joint market up and down if  $F_Z$  or  $F_\varepsilon$  is asymmetric (Oh and Patton, 2017a). Applying a non-normal distribution to the model from equation (14), the systemic factor may have less weight on the upper tail than on the lower tail whereas  $\varepsilon$  is symmetrically distributed (tail index of  $\varepsilon$  equals lower tail index of  $Z$ ). Then, the upper tail dependence from equation (10) equals zero and the lower tail dependence from equation (9) is positive.

Referring to the one-factor copula model from equation (14),  $F_Z = \text{Skew } t(\nu, \lambda)$  and  $F_\varepsilon = t(\nu)$ , the tail indices of  $Z$  and  $\varepsilon$  equal the degrees of freedom  $\nu$  and the constants from equation (9) and (10) can be calculated as follow:

$$A_Z^L = \frac{bc}{\nu} \left( \frac{b^2}{(\nu-2)(1-\lambda)^2} \right)^{\frac{-(\nu+1)}{2}}, \quad A_Z^U = \frac{bc}{\nu} \left( \frac{b^2}{(\nu-2)(1+\lambda)^2} \right)^{\frac{-(\nu+1)}{2}}$$

$$A_\varepsilon^L = A_\varepsilon^U = \frac{c}{\nu} \left( \frac{1}{\nu-2} \right)^{\frac{-(\nu+1)}{2}}$$

where the parameters of the defined distribution are used for  $a = 4\lambda c(\nu-2)(\nu-1)$ ,  $b = \sqrt{1+3\lambda^2-a^2}$  and  $c = \Gamma(\frac{\nu+1}{2})/(\Gamma(\frac{\nu}{2})\sqrt{\pi(\nu-2)})$  (Oh and Patton, 2017a).

Apart from the copula-implied tail dependence measures above, the tail dependencies can also be obtained with a non-parametric approach. These measures are widely known as the empirical tail dependence coefficients. Schmidt and Stadtmüller (2006) propose to estimate the coefficients by two nonparametric estimation methods, either by using the empirical tail copula or based on the stable tail-dependence function. Since these non-parametric estimators apply empirical distribution functions to model the marginal distributions, it circumstanced the lack of possible misidentification arising from a wrong parametric specification. The non-parametric fit provides also more flexibility by dropping

the restrictive fashion of a parametric model.

$$\hat{\tau}_{ij}^L := \frac{n}{k} C\left(\frac{kx_i}{n}, \frac{kx_j}{n}\right) \approx \frac{1}{k} \sum_{t=1}^n \mathbf{I}(R_i^{(t)} \leq kx_i, R_j^{(t)} \leq kx_j) \quad (11)$$

$$\hat{\tau}_{ij}^U := \frac{n}{k} \tilde{C}\left(\frac{kx_i}{n}, \frac{kx_j}{n}\right) \approx \frac{1}{k} \sum_{t=1}^n \mathbf{I}(R_i^{(t)} > n - kx_i, R_j^{(t)} > n - kx_j) \quad (12)$$

where  $\tilde{C}$  is the empirical survival copula with  $\tilde{F}_i = 1 - F_i$ ,  $k$  is a threshold parameter and  $R$  denotes the rank of the underlying data  $x$ . The very right sides in equation (11) and (12) show in an approximative framework two rank-order-statistics based on a modified empirical tail copula. Further details can be found in Schmidt and Stadtmüller (2006).

### 2.3 The factor copula model

Portfolio models can be distinguished between reduced form models and structural models to which factor models belong. Factor models have become frequently applied in several sciences. These models can capture agent's shared behavior through joint common factors. When combining variables to a lower number of common factors, factor models are powerful tools for dimension reduction. Moreover, factor copulas models outperform other copula classes in terms of their high usefulness for copula parameter estimation under the curse of dimensionality and time complexity. Introduced by Krupskii and Joe (2013) factor copulas are explained as a copula framework applied on a factor structure which enhances high-dimensional estimations and lowers the computational burden due to a reduced number of parameters (Krupskii and Joe, 2013). This more flexible model setting allows to fit dependence structure more adequately than linear dependence measures since the copula framework can serve for non-linear and varying dependence structures among the variables and the common factors.

In general, the multivariate factor copula model presumes a linear dependence structure of  $d$  observed variables  $Z$  on  $p$  conditional factors  $W$ :

$$Z_i = \sum_{k=1}^p \alpha_{i,k} W_k + \varepsilon_i \quad (13)$$

where  $W \sim iid F_W(\gamma_W)$ ,  $\varepsilon_i \sim iid F_\varepsilon(\gamma_\varepsilon)$  and  $W_k \perp \varepsilon_i \forall i, k$  based on  $d + p$  latent variables (Oh and Patton, 2017a). Thereby, the common factors  $W$  are assumed to be independent and identically distributed. The independence assumption is required by the numerical optimization procedure when estimating the copula density in section 3. The latter assumption simplifies the estimation by reducing the number of parameters being obtained

by the numerical integration method. Moreover, the parameters of the common factors' distributions are assumed to be equal to further mitigate the numerical integration efforts. However in this work, the factor model from equation (13) (Krupskii and Joe, 2013) reduces to one factor  $p = 1$ :

$$Z_i = \alpha_i W + \sqrt{1 - \alpha_i^2} \varepsilon_i \quad (14)$$

where all assumptions on the distributions  $F_W$  and  $F_\varepsilon$  from the multiple factor case of equation (13) stay valid and the vector of the factor copula parameters is defined as  $\theta_C = (\alpha_1, \dots, \alpha_d, \gamma'_W, \gamma'_\varepsilon)$ . Now, in order to construct the copula and without any loss of generality, the margins are defined as i.i.d. random variables and follow  $Uniform(0, 1)$ . Hence, the random variables  $Z_i$  are transformed to conditionally independent uniform random variables  $u_i \equiv F_{Z_i}(Z_i)$  given the uniform representation of the common factor  $v \equiv F_W(W)$ . Then, using  $Z_i = F_{Z_i}^{-1}(u_i)$  and applying the factor model on the copula function from equation (2), the factor copula writes as follows:

$$\begin{aligned} C_V(u_1, \dots, u_d) &= F(Z_1, \dots, Z_d) \\ &= \int_0^{1^p} F_{Z|v_1, \dots, v_p}(Z|v_1, \dots, v_p) dF_v(v_1) \dots dF_v(v_p) \end{aligned} \quad (15)$$

where  $F_{Z|V}$  represents the conditional joint cdf of the vector  $U$  on the vector  $V$ . After transforming the  $p$  independent variables to uniform random variables, any conditional independence model is able to fit this form. Considering the univariate factor case  $p = 1$  and under the assumption of independent  $U \equiv (u_1, \dots, u_d)^T$ , it follows:

$$\begin{aligned} C_v(u_1, \dots, u_d) &= \int_0^1 \prod_{i=1}^d F_{Z_i|v}(Z_i|v) dF_v(v) \\ &= \int_0^1 \prod_{i=1}^d C_{F_Z(Z_i)|v}(F_Z(Z_i)|v) dv \\ &= \int_0^1 \prod_{i=1}^d C_{u_i|v}(u_i|v) dv \end{aligned} \quad (16)$$

Since  $u_i, v$  are uniform random variables and  $C_{u_i, v}$  and  $c_{u_i, v}$  are their joint cdf and density, it holds that  $F_{u_i|v}(u_i|v) = C_{u_i|v}(u_i|v) = \frac{\partial C_{u_i, v}(u_i, v)}{\partial v}$  which is used to derive the second equation above. Equation (16) denotes the factor copula model consisting of a sequence of bivariate copulas which link the random variables  $u_i$  to the common factor  $v$ . Noting that  $\frac{\partial C_{u_i|v}(u_i|v)}{\partial u_i} = \frac{\partial^2 C_{u_i, v}(u_i, v)}{\partial u_i \partial v} = c_{u_i, v}(u_i, v)$  and plugging equation (3) into equation (16),



the one-factor copula density becomes:

$$c(u_1, \dots, u_d) = \int_0^1 \prod_{i=1}^d c_{u_i, v}(u_i, v) dv \quad (17)$$

According to equation (4) all information about dependence are solely explained by means of the copula density that due to the just derived one-factor copula equation (17) consists of  $d$  bivariate linking copulas. More details can be found in Krupskii and Joe (2013). Now, the conditional pair copula will be derived from equation (16) and using the factor structure from equation (14):

$$\begin{aligned} C_{u_i|v}(u_i|v) &= F_{u_i|v}(u_i|v) \\ &= F_{\varepsilon_i} \left( \frac{F_{Z_i}^{-1}(u_i) - \alpha_i F_W^{-1}(v)}{\sqrt{1 - \alpha_i^2}} \right) \end{aligned} \quad (18)$$

which is the general expression for the conditional independence structure of the uniform margins within the factor model. That is,  $u_i$  are independent with conditioning variable  $v$  (McNeil et al., 2015). Based on the one-factor model from equation (14) the dependence structure can be modeled in various ways depending on the distributions of the common factor  $W$  and the idiosyncratic component  $\varepsilon$ . Note that the copula is only given in closed form solution for  $F_W$  and  $F_\varepsilon$  being normally distributed. The resulting multivariate normal distribution has been used by Lu et al. (2015) and Krupskii and Joe (2013). The closed form solution for normally distributed random variables is a Gaussian copula. Thus, the dependence structure is determined by the choices of distributions for  $W$  and  $\varepsilon$ .

Choosing instead heavy tail distributions and an asymmetric one for the common factor enable tail and asymmetric dependence which will be useful for the later analysis. A  $t$ -copula, for instance, enables joint heavy tails and a higher probability of joint extreme events in comparison to a Gaussian framework. Oh and Patton (2017a) provided extensive research on factor copula models and show the high degree of adaptability of this copula class by using normal,  $t$  and *Skew t* marginal distributions to model different dependence structures such as asymmetric and tail dependence. Chen and Nasekin (2017) applied a double- $t$  copula to a network-based factor model. In contrast to the Gaussian copula framework, heavy tail and asymmetric factor copulas do not provide a closed form solution. Considering such distribution for parameter estimation on equation (18), the solution for distribution  $F_{Z_i}$  is then to compute numerically via convolution of  $\alpha_i W$  and  $\sqrt{1 - \alpha_i^2} \varepsilon_i$  (Chen and Nasekin, 2017). According to Borak et al. (2011), the double generalized hyperbolic copula framework suggested by McNeil et al. (2015) and the double

normal inverse Gaussian copula framework modeled by Kalemanska et al. (2007), which both belong to the class of generalized hyperbolic distributions, yield the best fit for financial data. However, modeling tail dependence with generalized hyperbolic distributions demands restrictive assumptions on the parameters. Apart from the parametric family of elliptical copulas, Archimedean parametric copulas have been analyzed in factor settings by recent research from Granger et al. (2006), He and Gong (2009) and Shamiri et al. (2011) who studied approaches on Gumbel, Clayton and Frank copulas. Although, these Archimedean copulas have the properties of tail dependence and asymmetry and hence provide a possibly good fit to financial data, they are quite restrictive in high-dimensional frameworks. That is because the dependence between all random variables can only be determined by one or two parameters.

This work considers the *Skew t - t* factor copula proposed by Oh and Patton (2017a) in order to enable tail dependence and asymmetry and to provide a good fit to the data. The financial data are stock price returns of 28 banks which are weighted according to the banks' sizes to construct a system portfolio as representation of the common factor. To obtain a measure for systemic risk driven by idiosyncratic risk, the returns of each bank  $y_{i,t}$  are approximated by a GARCH(1, 1) model to isolate the idiosyncratic component. This component is cleaned from the GARCH effect by using each bank's zero mean returns to exclude the market impact and by standardizing the distribution within the GARCH(1, 1) framework (Fantazzini, 2008):

$$y_{i,t} = a_i + b_i y_{m,t} + \varepsilon_{i,t} \quad (19)$$

$$h_{i,t}^2 = \omega_i + \beta_i h_{i,t-1}^2 + \alpha_i \varepsilon_{i,t-1}^2 \quad (20)$$

$$Z_{i,t} = \sqrt{\frac{\nu_i}{h_{i,t}^2(\nu_i - 2)}} \varepsilon_{i,t} \sim t_{\nu_i, \lambda_i} \quad (21)$$

where firstly, in the upper equation the bank-specific returns are regressed on the market returns. Secondly, the regressions' residuals are controlled for the GARCH effect with one lag for both elements, the autoregressive and the conditional variance part. Thus, the obtained idiosyncratic residual returns and the systemic residual returns are presumed to be *iid* random variables following a standardized *t* and a *Skew t* distribution (Chen and Nasekin, 2017). The respective variances within the factor copula model are set to one (Oh and Patton, 2017a). Tail and asymmetric dependence are ensured by applying a *Skew t - t* factor copula model.

First of all, the *Skew t* density is presented (Hansen, 1994):

$$f(W|\nu, \lambda) = \begin{cases} bc \left( 1 + \frac{1}{\nu-2} \left( \frac{bz+a}{1-\lambda} \right)^2 \right)^{-\frac{\nu+1}{2}} & z < -\frac{a}{b} \\ bc \left( 1 + \frac{1}{\nu-2} \left( \frac{bz+a}{1+\lambda} \right)^2 \right)^{-\frac{\nu+1}{2}} & z \geq -\frac{a}{b} \end{cases} \quad (22)$$

where

$$a = 4\lambda c \left( \frac{\nu-2}{\nu-1} \right), \quad b^2 = 1 + 3\lambda^2 - a^2, \quad c = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})} \quad (23)$$

$\Gamma$  is the gamma function and the parameters of the *t* distribution are given by  $\nu$  and  $\lambda$  which are the degrees of freedom and the skewness parameter, respectively. By definition, it holds that  $\nu \in (2, \infty)$  and  $\lambda \in (-1, 1)$ . The *Skew t* distribution has mean zero and unit variance. For increasing degrees of freedom  $\nu$ , the distribution approximates to the normal distribution and the probability of extreme co-movements decreases. The density function is left-skewed for  $\lambda < 0$ , symmetric for  $\lambda = 0$  and right-skewed for  $\lambda > 0$ . For  $\lambda = 0$  and  $\nu \neq +\infty$  it becomes the standardized *t* distribution and for the extreme case of infinitely large degrees of freedom the *t* distribution converges to the normal distribution. Keeping  $\nu = +\infty$  but having non-zero skew  $\lambda \neq 0$  it renders in a skewed normal distribution.

For the sake of simplicity, the degrees of freedom are equal for  $W$  and  $\varepsilon_i$  and are determined by the systemic factor. Hence, for a *Skew t* – *t* copula model with underlying factor structure of equation (14) and equal  $\nu$  for  $F_W$  and  $F_\varepsilon$  requires to estimate  $d+2$  parameters. That is,  $d$  copula parameters and one degree of freedom and one skewness parameter.

Then, equation (2) gives the multivariate *t*-copula of the multivariate *t*-distribution:

$$C_\nu^t(u_1, \dots, u_d) = T_\nu^d(T_1^{-1}(u_1), \dots, T_d^{-1}(u_d)) \quad (24)$$

where  $T_\nu^d$  is the multivariate *t*-distribution and  $T_\nu^{-1}$  is the inverse univariate cumulative distribution function.

The pair copula representation in equation (18) comes from the specific case of a Gaussian copula framework in which the derivation is independent from the factor copula parameter  $\alpha$  and hence in a Gaussian framework  $\alpha$  denotes nothing more than the linear correlation coefficient (Krupskii and Joe, 2013). This is given by the convolution stability of the Gaussian function. The solution to the Gaussian pair copula formulation would then result analytically since the sum of two Normal variables gives a Normal variable again. While for the Gaussian formulation (14) remains the same, this simple representation does not hold anymore in this *Skew t* – *t* model though and the distribution is neither stable under convolution nor provides any analytical solution. The distribution then becomes

a weighted sum of two unit-variance  $t$ -distributed random variables which need to be standardized such as  $W \doteq W(\nu)\sqrt{\frac{\nu-2}{\nu}}$  and  $\varepsilon_i \doteq \varepsilon_i(\nu)\sqrt{\frac{\nu-2}{\nu}}$  for a double  $t$  model. Vrina (2009), Kolman (2014) and Choroś-Tomczyka et al. (2012) express the conditional pair copula distribution for a double  $t$  one-factor model as follows:

$$C_{u_i|v}(u_i|v) = T_\nu \left[ \frac{Z_i - \alpha_i \sqrt{\frac{\nu-2}{\nu}} W}{\sqrt{1 - \alpha_j^2} \sqrt{\frac{\nu-2}{\nu}}} \right] \quad (25)$$

where  $Z_i = T_\nu^{-1}(u_i)$  and  $W = T_{\nu,\lambda}^{-1}(v)$ . Furthermore, with respect to the skew, Chan and Kroese (2010) present a one-factor copula model expression for a *Skew  $t$  -  $t$*  model. The *Skew  $t$  -  $t$*  model within a factor copula framework can also be expressed in terms of  $W \sim iid N(0, 1)$  and  $\varepsilon_i \sim iid N(0, 1)$ :

$$Z_i = \left( \alpha_i W + \lambda S + \sqrt{1 - \alpha_i^2} \varepsilon_i \right) \eta^{-1} \quad (26)$$

with

$$S \sim TN \left( -\sqrt{\frac{2}{\pi}}, 1 \right) \quad (27)$$

where  $TN(\mu, \sigma^2)$  is a truncated normal distribution with left truncated mean  $-\sqrt{\frac{2}{\pi}}$ .  $\lambda > 0$  then includes the right-skewed property to the distribution and  $\eta^{-1}$  is nothing more than a mathematical construct that brings the  $t$ -distributional property into the model such that  $\eta^2 \sim \Gamma(\frac{\nu}{2}, \frac{\nu}{2})$ . Anyway, this component can also be understood as shock term which triggers many simultaneous defaults for small  $\eta$ . This setting under a double  $t$  framework is also considered by Franke et al. (2004) and Chan and Kroese (2010). Due to the setting of equation (26), Chan and Kroese (2010) show that the distribution of  $Z_i$  becomes asymmetric and the skewness is induced by the parameter  $\lambda$  whereas the mean of  $Z_i$  is not affected by the term  $\lambda S$ . Oh and Patton (2017a), Embrechts et al. (2005) and Lin et al. (2013) however suggest to include the skew into the factor representation from equation (14) such as:

$$Z_i = \alpha_i \sqrt{\frac{\nu-2}{\nu}} W + \lambda \sqrt{\frac{\nu-2}{\nu}} \tilde{S} + \sqrt{(1 - \alpha_i^2) \frac{\nu-2}{\nu}} \varepsilon_i \quad (28)$$

where  $\tilde{S} = \frac{\nu}{\mathcal{V}}$  is scalar independent of  $W$  with  $\mathcal{V} \sim \chi_\nu^2$  and thus  $\tilde{S} \sim Inv \chi_\nu^2$  (Embrechts et al., 2005).<sup>1</sup> Due to this simple formulation in equation (28) the double  $t$  copula model becomes generalized to allow for an asymmetric distribution triggered by the skewness

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<sup>1</sup>Note that the inverse chi-squared distribution is a special case of the inverse Gamma distribution from  $\eta^2$  in equation (26) with some restrictions on the Gamma distribution parameters. For more details on the extension of a double  $t$  to a *Skew  $t$  -  $t$*  factor model, see Embrechts et al. (2001), Frey and McNeil (2001) and Oh and Patton (2017a).

parameter  $\lambda$  (Chan and Kroese, 2010). Applying the distributional assumptions on the one-factor structure of equation (28) with normal margins, the conditional copula (25) becomes:

$$C_{u_i|v}(u_i|v) = T_\nu \left[ \frac{Z_i - \lambda \sqrt{\frac{\nu-2}{\nu}} \tilde{S} - \sqrt{\frac{\nu-2}{\nu}} \alpha_i W}{\sqrt{1 - \alpha_j^2} \sqrt{\frac{\nu-2}{\nu}}} \right] \quad (29)$$

By equation (16) and (29) it follows:

$$C_v(u_1, \dots, u_d) = \int_{-\infty}^{\infty} \prod_{i=1}^d \left\{ T_\nu \left[ \frac{Z_i - \lambda \sqrt{\frac{\nu-2}{\nu}} \tilde{S} - \sqrt{\frac{\nu-2}{\nu}} \alpha_i W}{\sqrt{1 - \alpha_j^2} \sqrt{\frac{\nu-2}{\nu}}} \right] \right\} t_\nu(W) dW \quad (30)$$

For the sake of simplicity, the following sections will stick to the expression introduced by equation (14) to represent the *Skew t-t* model under the distributional assumptions of  $W \sim iid T_W(\nu, \lambda)$ ,  $\varepsilon_i \sim iid T_\varepsilon(\nu)$  where  $W \perp \varepsilon_i \forall i, k$ . Therefore, Frey and McNeil (2001) also use the simplified setting of an exchangeable one-factor model for various distributions. Hence, for the further analysis, the equation (18) is used. This representation for a *Skew t-t* framework is also chosen by Yang et al. (2009) and Oh and Patton (2017a) who use an exchangeable one-factor model representation for the sake of comparison between the different distributional assumptions in their extensive work on factor copula models. Azzalini and Capitanio (2003) also apply the notation of the one-factor copula model structure from equation (14) on skewed elliptical distribution frameworks. Bluhm and Wagner (2011) employ a mixed distribution approach on the same one-factor model formulation.

The equation (30) has, as already stated above, no closed form solution and the model implementation is mainly dependent on an accurate integral computation. The copula parameters are estimated by maximum likelihood estimation employing the Gauss-Legendre quadrature for numerical integration for the copula density estimation. The next section will enlarge upon the factor copula density estimation by numerical integration method.

### 3 Estimation of the factor copula model

In general, the estimation of a multivariate distribution based on a copula can be accomplished in only one step that includes the estimation of the copula parameters  $\theta_C$  and the estimation of the margins  $F_i$  with  $i = 1, \dots, d$ . However, as mentioned in section 2.1, Sklar's Theorem also allows for multi-stage estimation which provides advantages for the computation in terms of high-dimensionality. The applied maximum likelihood estimations furthermore can be embedded in parametric and nonparametric models for the margins. Thereby, the entire distribution is considered for parametric models, that are the full maximum likelihood estimation and the inference for margins method which are one-step and two-step procedures, respectively. The one-step simultaneous estimation procedure achieves consistent and efficient estimators but calls for high numerical complexity under high dimensionality since the simultaneous method jointly estimates the marginal and the copula parameters. Semi- and nonparametric estimation methods for the margins are quite suitable if the interest is merely on the dependence structure and to avoid a parametric restriction to the unknown marginal distributions. Then, the multi-stage semiparametric method, that is the pseudo maximum likelihood, also canonical maximum likelihood, obtains nonparametric estimates for the marginal distributions by applying empirical distribution functions and thus, enables the estimates to be independent of restrictive parametric families. After that, the dependence structure between the margins is obtained by using a parametric copula estimation for the uniformly transformed pseudo sample (Kim et al., 2007). As shown by results of Genest et al. (1995) the simultaneous estimation of the full maximum likelihood is preferred over two-stage solutions, although the pseudo maximum likelihood estimator becomes consistent and asymptotically efficient under certain conditions.

In this framework, the estimation of the factor copula model is carried out with a multi-stage maximum likelihood that requires the log-likelihood function of the one-factor copula model to become decomposed into two components. This is necessary to enhance the computational performance in high-dimensional problems. Next the estimators are presented before the standard errors will be obtained. Finally, a numerical integration and optimization procedure for maximum likelihood is applied to achieve an expression for the factor copula density (17). Thereby, the estimation of the cumulative distribution function proceeds the inverse distribution function estimation which can be calculated afterwards numerically (Vrins, 2009). Now, suppose under the copula density from equation (17) belongs to a parametric family  $C = \{C_\theta, \theta_C \in \Theta_C\}$  and considering equation (4) and the parameter vector  $\theta = (\theta_{m,1}, \dots, \theta_{m,d}, \theta_C)' \in \mathbb{R}^{d+1}$ , the likelihood function is written as follows:

$$L(y_1, \dots, y_n; \theta) = \prod_{t=1}^n f(y_{1,t}, \dots, y_{d,t}; \theta_{m,1}, \dots, \theta_{m,d}, \theta_C) \quad (31)$$

Now, the likelihood function (31) must be decomposed into two components, the likelihood contributions from the marginal distributions and from the dependence structure (Choroś et al., 2010). The decomposed log-likelihood function is given by:

$$l(y_1, \dots, y_n; \theta_m, \theta_C) = l_m(\theta_m) + l_C(\theta_C, \theta_m) \quad (32)$$

which can be rewritten as:

$$\begin{aligned} l(y_1, \dots, y_n; \theta_m, \theta_C) &= \sum_{t=1}^n \sum_{i=1}^d \log f_i(y_{i,t}; \theta_{m,i}) \\ &+ \sum_{t=1}^n \log c(F_1(y_{1,t}; \hat{\theta}_{m,1}), \dots, F_d(y_{d,t}; \hat{\theta}_{m,d}); \theta_C) \end{aligned} \quad (33)$$

where the estimation of the parameters for the uniform margins takes place at first stage and for the copula parameters at second stage with the marginal parameters fixed at the estimates from the first estimation stage. The log-likelihood for the marginal distributions is further decomposed into  $d$  log-likelihood expressions for  $d$  margins. The resulting estimates for  $\theta_m$  are required for the copula parameter estimation since the probability integral transforms are derived using the marginal parameters. The first stage estimation to obtain the  $d$  parameters of the marginal distributions is given by:

$$\hat{\theta}_{m,i} = \underset{\theta_{m,i}}{\operatorname{argmax}} l_{m,i}(\theta_{m,i}) \quad (34)$$

The factor copula density in the second stage has no analytical solution for the distributional assumptions made in this framework. Hence, the log-likelihood of the copula model is solved via a numerical integration method which will be discussed later in section 3.2. Given the estimated marginal parameters  $\hat{\theta}_m = (\hat{\theta}_{m,1}, \dots, \hat{\theta}_{m,d})'$  the dependence parameters solve the pseudo log-likelihood function  $l_C(\theta_C, \hat{\theta}_m)$  at second stage:

$$\hat{\theta}_C = \underset{\theta_C}{\operatorname{argmax}} l_C(\theta_C, \hat{\theta}_m) \quad (35)$$

Except for the particular case of a multivariate Gaussian copula with normal marginal distributions, the inference for margins estimators differ from the efficient and consistent maximum likelihood estimator (Choroś et al., 2010). Although, this two-stage estimation procedure results in less efficient estimators than the full maximum likelihood method,

Joe (1997) argued that the modest loss in efficiency is tolerable compared to the benefits from the material reduction of the computational efforts.

### 3.1 The factor copula density

By applying non-gaussian marginals to the factor copula approach the likelihood function will most likely not yield a closed form solution for the factor copula density of equation (16) and (18). Oh and Patton (2016) derived a computationally feasible representation for the copula density:

$$c(u_1, \dots, u_d; \theta) = \frac{f_{\mathbf{Z}}(F_{Z_1}^{-1}(u_1), \dots, F_{Z_d}^{-1}(u_d))}{f_{Z_1}(F_{Z_1}^{-1}(u_1)) \cdot \dots \cdot f_{Z_d}(F_{Z_d}^{-1}(u_d))} \quad (36)$$

Based on an one-factor model, the component of equation (36) can be found by the derivation of  $f_{\mathbf{Z}}(Z_1, \dots, Z_d)$ ,  $F_{Z_i}^{-1}(u_i)$  and  $f_{Z_i}(Z_i)$  which only requires to presume the independence between the common factor  $W$  and the idiosyncratic component  $\varepsilon_i$ . Then, the joint density, the marginal distribution and density of  $Z_i$  are given by  $f_{Z_i}$ ,  $F_{Z_i}$  and  $f_{\mathbf{Z}}$ , respectively and can be derived using (16) and (18):

$$f_{Z_i|W}(Z_i|W) = f_{\varepsilon_i} \left( \frac{Z_i - \alpha_i F_W^{-1}(v)}{\sqrt{1 - \alpha_i^2}} \right), \quad (37)$$

$$F_{Z_i|W}(Z_i|W) = F_{\varepsilon_i} \left( \frac{Z_i - \alpha_i F_W^{-1}(v)}{\sqrt{1 - \alpha_i^2}} \right), \quad (38)$$

$$f_{\mathbf{Z}|W}(Z_1, \dots, Z_d|W) = \prod_{i=1}^d f_{\varepsilon_i} \left( \frac{Z_i - \alpha_i F_W^{-1}(v)}{\sqrt{1 - \alpha_i^2}} \right). \quad (39)$$

To obtain the marginals, the conditional distributions are now integrated w.r.t. the common factor  $W = F_W^{-1}(v)$  (Oh and Patton, 2017b):

$$f_{Z_i}(Z_i) = \int_{-\infty}^{\infty} f_{\varepsilon_i} \left( \frac{Z_i - \alpha_i W}{\sqrt{1 - \alpha_i^2}} \right) dW, \quad (40)$$

and analogously for  $F_{Z_i}(Z_i)$  and  $f_{\mathbf{Z}}$ :

$$F_{Z_i}(Z_i) = \int_{-\infty}^{\infty} F_{\varepsilon_i} \left( \frac{Z_i - \alpha_i W}{\sqrt{1 - \alpha_i^2}} \right) dW, \quad (41)$$

$$f_{\mathbf{Z}}(Z_1, \dots, Z_d) = \int_{-\infty}^{\infty} \prod_{i=1}^d f_{\varepsilon_i} \left( \frac{Z_i - \alpha_i W}{\sqrt{1 - \alpha_i^2}} \right) dW. \quad (42)$$



Krupskii and Joe (2013) stated that the integrand could be unbounded since for many parametric copula families the copula density becomes infinitely large as the uniform random variables  $u_i, v$  approach the boundaries 0 or 1. Joe (2014) argued in this regard that the integrand is an integrable function. Since the numerical integration method will estimate the inverse distribution of the uniform random variable, it writes  $Z_i = F_{Z_i}^{-1}(u_i)$  with the uniform representation of the common factor  $v = F_W(W)$ :

$$f_{Z_i}(Z_i) = \int_0^1 f_{\varepsilon_i} \left( \frac{F_{Z_i}^{-1}(u_i) - \alpha_i F_W^{-1}(v)}{\sqrt{1 - \alpha_i^2}} \right) dv, \quad (43)$$

$$F_{Z_i}(Z_i) = \int_0^1 F_{\varepsilon_i} \left( \frac{F_{Z_i}^{-1}(u_i) - \alpha_i F_W^{-1}(v)}{\sqrt{1 - \alpha_i^2}} \right) dv, \quad (44)$$

$$f_{\mathbf{Z}}(Z_1, \dots, Z_d) = \int_0^1 \prod_{i=1}^d f_{\varepsilon_i} \left( \frac{F_{Z_i}^{-1}(u_i) - \alpha_i F_W^{-1}(v)}{\sqrt{1 - \alpha_i^2}} \right) dv. \quad (45)$$

Inserting the derived joint density  $f_{\mathbf{Z}}$ , marginal distribution  $F_{Z_i}$  and density  $f_{Z_i}$  into the copula density (36), the resulting implied copula density is now computationally feasible for the numerical integration procedure to achieve solutions for the maximum likelihood estimation:

$$c(u_1, \dots, u_d; \theta) = \frac{\int_0^1 \prod_{i=1}^d f_{\varepsilon_i} \left( \frac{F_{Z_i}^{-1}(u_i) - \alpha_i F_W^{-1}(v)}{\sqrt{1 - \alpha_i^2}} \right) dv}{\int_0^1 f_{\varepsilon_1} \left( \frac{F_{Z_1}^{-1}(u_1) - \alpha_1 F_W^{-1}(v)}{\sqrt{1 - \alpha_1^2}} \right) dv \cdot \dots \cdot \int_0^1 f_{\varepsilon_d} \left( \frac{F_{Z_d}^{-1}(u_d) - \alpha_d F_W^{-1}(v)}{\sqrt{1 - \alpha_d^2}} \right) dv} \quad (46)$$

Although, the copula density spans over  $d$  dimensions, the numerical integration circumvents an high-dimensionality optimization by only taking the integral over the common factor. Thus, the integrals from the implied copula density with one common factor (46) are obtained from one-dimensional numerical integration on the interval  $[0, 1]$  (Oh and Patton, 2017b).

### 3.2 Numerical Integration

Numerical integration is a widely encountered problem in mathematical science. The general concept is to approximate a definite integral by a weighted sum of function values at points within the interval of integration. Thereby, these basic quadrature methods can be divided into two wide classes regarding the distance between the data points. While the first class of methods, these are Newton-cotes formulas, distributes the data points equidistantly, the second class of methods distributes the data points unequally over the

interval. The latter methods belong to more efficient Gaussian quadrature techniques which are based on the orthogonal polynomial approach to functional approximation of the following quadrature problem:

$$\int_{-\infty}^{\infty} f(x)dx \simeq \sum_{k=0}^q \omega_k f(x_k) \quad (47)$$

where the quadrature nodes are denoted by  $x_k \in [a, b]$ , their quantities by  $q$  and the corresponding quadrature weights by  $w_k$ . Hence, this approach mainly differ by the nodes at which the function is evaluated and by the weights which are assigned to each function value. The precision of the approximation is subject to the choice of the quadrature nodes  $x_k$  and weights  $\omega_k$ . In general, Gaussian quadrature chooses the weights and nodes such that the approximation is exact for a low order polynomial  $f$ . In fact, the approximation error becomes strictly zero for every polynomial  $f$  being of order less than  $2q$  (Vrins, 2009). The most common Gaussian quadrature methods are the Gauss-Chebyshev, the Gauss-Legendre, the Gauss-Hermite and the Gauss-Laguerre formula.

Considering such a quadrature formula for the numerical integration method applied on the equations (43), (44) and (45), the problem takes a one-dimensional fashion whereas it becomes a multivariate numerical integration for multi-factor models. In this case, multivariate integrals need to be computed and the nodes and weights are obtained by Kronecker product. Krupskii and Joe (2013) and Oh and Patton (2017b) used Gauss-Legendre quadrature for the numerical integration and optimization for the maximum likelihood estimation. Vrins (2009) computed Gauss-Hermite quadratures to a problem on a double- $t$  factor copula framework. However, the Gauss-Hermite quadrature might obtain non-finite error bounds for such framework which contains integrands taking unbounded values. The Gauss-Legendre quadrature provides bounded approximation errors (Kahaner et al., 1989). Oh and Patton (2017b) compared numerical integration procedures for a normal factor copula with 10, 50 and 150 nodes with the closed-form likelihood results which relates to  $\infty$  nodes. The paper found that the accuracy for 50 and 150 nodes does not differ much from the analytical solution while the error increases for only a small number of nodes. Joe (2014) proposed a number of quadrature nodes per dimension of  $q = 20 - 30$  to be optimal with respect to a numerically stable maximum likelihood estimation. Chen and Nasekin (2017) found out empirically that  $q = 21$  maximizes the likelihood of their model.

Based on existing literature on numerical procedures for factor copula models using  $t$  distributions, this framework will apply the Gauss-Legendre quadrature to solve the numerical integration problem for the integrals of (43), (44) and (45). The quadrature nodes

are obtained by the roots of the Legendre polynomial which belongs to the family of orthogonal polynomials. This framework benefits from the use of orthogonal polynomials that enhance the computational speed and allow for high non-linearities. The roots of the orthogonal Legendre polynomial can be calculated by a root finding procedure, such as the Newton-Raphson method. This linear approximation of the function  $f(x)$  is performed by a succession of linear functions. Among the root finding procedures, the Newton Raphson method is often the fastest method and also easy to implement in a multidimensional setting. Then the weights are given by the obtained nodes and the Legendre polynomial (Judd, 1998).

Applied on this model, values of  $F_{Z_i}(u; \alpha_i)$  can be calculated by using the Newton-Raphson root finding method that runs the iteration until the latest obtained points are within a threshold of the previous points (Judd, 1998). The inverse cdf  $F_{Z_i}^{-1}$  must be estimated in order to obtain the solutions to the Gauss-Legendre quadrature. The numerical integration and optimization for maximum likelihood of equation (31) can now obtain the parameters  $\theta$  of the joint density  $c(u_{1,t}, \dots, u_{d,t}; \theta)$  from (36):

$$L(u_1, \dots, u_n; \theta) = \prod_{t=1}^n c(u_{1,t}, \dots, u_{d,t}; \theta) \quad (48)$$

For the estimation of the inverse cdf,  $x_k$  from equation (47) is distributed on a grid on the interval  $[x_{min}, x_{max}]$  and quadrature nodes are  $q = 21$ . The inverse of  $F_{Z_i}$  can be determined by evaluating  $F_{Z_i}$  for the uniform random variable  $u$  and the copula parameter vector  $\alpha_i$  at each point of the two grids each having 100 grid points on the intervals  $[0, 1]$  and  $[-1, 1]$ , respectively (Chen and Nasekin, 2017). Finally, to obtain all values for the inverse cdf, piecewise cubic interpolation is applied in each maximum likelihood estimation iteration. Since the values of  $F_{Z_i}^{-1}(u; \alpha_i)$  are only estimated once before the maximum likelihood estimation, it reduces essentially the computational burden.

## 4 Measuring systemic risk in a factor-copula framework

This analysis considers a weighted portfolio from the systemically important financial institutions to be an adequate representation for systemic risk. This section first presents a selection of popular and acknowledged risk measures. In further analysis, a scenario will be constructed which sets the model representation of the financial system under distress. Based on the stressed systemic factor, further risk measures will be derived to quantify systemic risk in specific institutions.

### 4.1 Value-at-Risk and Expected Shortfall

Firstly, the most common and applied measure for quantifying risk is presented by the Value-at-Risk which is widely used among financial regulation institutions and the private financial sector. In line with the literature the risk measures in this section are defined traditionally as estimates of the historical returns  $y_{i,t}$ . In this work however, the risk measures are taken based on the SIFI and system returns modeled by the one-factor copula model from equation (14). The VaR of the random variable  $y_{i,t}$  is given by the  $q$ -quantile of a return series:

$$P(y_{i,t} \leq VaR_q^i) = q \quad (49)$$

Hence, the VaR measures the maximal negative return of an underlying return series over a certain time period conditioned on a significance level  $q$ . There are several classifications to distinguish the estimation methods on VaR. The VaR estimation can be accomplished non- or parametrically, model-based or historically. Common methods estimate on basis of model-based approaches or by historical simulation. While analytical methods imposing distributional assumptions such as the Delta-Normal approach are simple to implement, they might lead to highly misspecified results. Its weakness mainly sources from the parametric fashion of setting a distributional assumptions on the underlying returns. The Monte-Carlo simulation also restricts the model to a parametric setting assuming a distribution on which a large-number simulation is run. The historical simulation belongs to the nonparametric approaches since it considers the empirical distribution and is associated with less computational efforts than the time-consuming Monte-Carlo simulation (Hull, 2006).

In this work, the VaR, based on equation (49), is a quantile of empirical distributions of return series. The drawback of the VaR as risk measure lies in the missing information on the values which exceed the VaR estimate. The Expected Shortfall addresses this issue and has become a popular risk measure which might deliver more adequate information on

the risk of a return series of financial institutions than the VaR.<sup>2</sup> The Expected Shortfall, also referred to as Conditional Value at Risk or Tail Loss, is defined as the mean over the returns once the VaR is exceeded and hence can be written as follows (Hull, 2006):

$$ES_q^i = \mathbb{E}[y_{i,t} | y_{i,t} < VaR_q^i] \quad (50)$$

This alternative to the VaR is subject to a higher sensitivity to the tail of the distribution. Anyway, both risk measures only give an isolated measurement of risk of each bank separately. This work focuses on measuring systemic risk in financial institutions and therefore, extended risk measures will be introduced.

## 4.2 Systemic risk measures

Apart from the above isolated risk measure for individual banks, conditional risk measures are introduced which can be estimated using conditional distributions to identify the risk exposure of banks to the system and the impact from banks on the system.

### 4.2.1 CoVaR

CoVaR means "conditional, contagion or comovement" VaR and is defined as the VaR of the whole financial system given institution  $i$  being under stress. Taking the difference between the CoVaR conditional on bank  $i$ 's distress and the CoVaR conditional on the median state of bank  $i$  results in  $\Delta\text{CoVaR}$ , the marginal impact of institution  $i$  to the financial system when institution  $i$  moves from median state into distress. This measure was introduced by Adrian and Brunnermeier (2008) who claim the existing risk measures, on the one hand, to neglect negative spillover effects by institutions on the system and on the other hand, to render in misleading results while mainly focusing on simultaneous price evolutions. Based on this criticism, their proposed systemic risk measures address the issue of the backward-looking fashion of the VaR by employing idiosyncratic elements such as the institutions' sizes, leverages and equity market beta.

Moreover, the  $\Delta\text{CoVaR}$  extends the usual risk measures which only quantify individual risk by measuring the impact on a predefined financial system. They argue that relying on potentially misleading risk estimation based on isolated risk measures of institutions misspecifies the true institution-inherent risk and allows these institutions to excessively accumulate risk. However, the VaR measure might last for the case in which the CoVaR measures run proportionally to the VaR. According to Adrian and Brunnermeier (2008),

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<sup>2</sup>Moreover, in contrast to the ES the VaR is not a coherent risk measure since it lacks in the subadditivity condition. That is, the VaR of the sum of a selection of portfolios is higher than the sum of the single portfolio VaRs. For more on the conditions for coherent risk measures, see Franke et al. (2004).

there is no empirical evidence that the  $\Delta\text{CoVaR}$  can be linked to a scaled VaR estimate. In contrast, they show the weak dependence between individual VaR estimates and their  $\Delta\text{CoVaR}$  measure. Apart from the contribution to the systemic risk, this risk measure also provides the flexibility to analyze the institution  $i$ 's risk impact on other institutions which directly addresses to the widely discussed issue of the interconnectness in the financial system. More precise, the CoVaR of Adrian and Brunnermeier (2008) is given by:

$$P(y_{j,t} \leq \text{CoVaR}_q^{j|\mathbb{C}(y_{i,t})} | \mathbb{C}(y_{i,t})) = q \quad (51)$$

where  $y_{j,t}$  denotes the returns of institution  $j$  (or the entire financial system) and  $\mathbb{C}(y_{i,t})$  some event of institution  $i$ .  $\mathbb{C}(y_{i,t})$  usually represents a stressed event of institution  $i$ , such as  $y_{i,t} = \text{VaR}_q^i$ . Adrian and Brunnermeier (2008) also analyze the case in which  $j$  stands for the financial system. Hence, the CoVaR measures the VaR of the financial system conditional on the bank specific returns being equal to the bank's VaR. The  $\Delta\text{CoVaR}$  of Adrian and Brunnermeier (2008) is then calculated by:

$$\Delta\text{CoVaR}_q^{j|i} = \text{CoVaR}_q^{j|y_{i,t}=\text{VaR}_q^i} - \text{CoVaR}_q^{j|y_{i,t}=\text{VaR}_{0.5}^i} \quad (52)$$

where  $\text{CoVaR}_q^{j|y_{i,t}=\text{VaR}_{0.5}^i}$  represents the VaR of the financial system conditional on the median state of institution  $i$ . Besides the quantile regression approach of Adrian and Brunnermeier (2008), this CoVaR measure was investigated in a comparative analysis by Benoit et al. (2013) and within a copula framework by Reboredo and Ugolini (2015). Related to the CoVaR measure between different institutions, Claessens and Forbes (2001) use a multivariate GARCH model to study volatility spillovers. Based on the idea of Hartmann et al. (2004) which provide research on contagion measures during crisis periods, Mainik and Schaanning (2014) choose another distress condition which addresses a more severe risk event for  $\mathbb{C}(y_{i,t})$  where  $y_{i,t} \leq \text{VaR}_q^i$ . While Mainik and Schaanning (2014) extend the CoVaR analysis of Adrian and Brunnermeier (2008), they show that a higher dependence parameter is associated with higher systemic risk. This is a result of the different approach for the conditioning distress event and was not found by the CoVaR approach of Adrian and Brunnermeier (2008). Girardi and Ergün (2013) provide an extensive research on  $\Delta\text{CoVaR}$  and their backtesting results for the period from 2000 until 2008 among the US financial sector suggest to include kurtosis and skewness into the model. Furthermore, Cao (2013) presents Multi-CoVaR to advance the systemic risk contribution measure for several institutions moving contemporaneously in distress. A further approach on Multi-CoVaR using time series processes has been implemented by recent research of Bernardi et al. (2013). Important contribution to the risk measure liter-

ature is provided by Engle and Manganelli (2004) who propose the CAViaR approach - a sort of dynamic setting of the CoVaR model using a GARCH process on the time variant tail distribution of return series. The literature on systemic risk rather focuses on the risk contribution of each institute to the financial system which is approached by  $\Delta\text{CoVaR}$ . While the effect of a market downturn requires to estimate the systemic contribution to each individual institution

#### 4.2.2 Exposure CoVaR

This work will continue with the CoVaR approach conditioning on the financial system being in distress to study the impact of a financial crisis on each institution. Hence,  $\Delta\text{CoVaR}_q^{i|y_{m,t}}$  of Adrian and Brunnermeier (2008) is called Exposure  $\Delta\text{CoVaR}$  with  $y_{m,t}$  being the market returns. The Exposure  $\Delta\text{CoVaR}$  thus gives the VaR of each institution  $i$  conditional on the financial system moving from median to distressed state:

$$\Delta\text{CoVaR}_q^{i|m} = \text{CoVaR}_q^{i|y_{m,t}=\text{VaR}_q^m} - \text{CoVaR}_q^{j|y_{m,t}=\text{VaR}_{0.5}^m} \quad (53)$$

This measure is more akin to the factor-copula model of this work which imposes the financial system returns as common systemic factor to all institutions. Since the Exposure  $\Delta\text{CoVaR}$  measures the impact of a market downturn on each institution, it can be seen as the individual institution's vulnerability to a switching state of the system which is one focus of this work.

Pagano and Sedunov (2014) provide an empirical analysis and show that the total systemic risk exposure of banks increases with the sovereign debt yields in Europe. Risk managers and regulators may utilize the Exposure  $\Delta\text{CoVaR}$  measure which is a complement for stress testing in individual institutions. Löffler and Raupach (2016) critically reviewed the imposed measures of Adrian and Brunnermeier (2008) and pointed out some caveats which come along with these risk measures contingent on the distributional assumption, the estimation method and the underlying data.

#### 4.2.3 Marginal Expected Shortfall

Based on the expected shortfall from equation (50), Acharya et al. (2017) introduce a measure which offer a linkage between losses of the entire financial system and the corresponding contribution from each institution. The marginal expected shortfall (MES) calculates the mean over the individual institution's returns conditional on the system

returns being below a certain threshold:

$$MES_q^i = E[y_{i,t} | y_{m,t} \leq VaR_q^m] \quad (54)$$

Hence, it measures the bank  $i$ 's losses when the financial system is in severe distress. Acharya et al. (2017) firstly apply the MES on institution-wide level which is particularly useful for institution-internal risk management that might aim to quantify the maximal potential losses of a single department occurring from a stressed state of the entire bank. Then, this MES can be scaled up to the entire financial system attempting to measure systemic risk and the exposure of each bank to the state of the financial system. Considering the bank  $i$ 's sizes relative to the entire financial system, the MES can also be expressed as the derivative of entire financial system's expected shortfall with respect to a bank  $i$ 's relative size  $\omega_i$  (Acharya et al., 2017):

$$\begin{aligned} MES_q^i &= \frac{\partial ES_q^m}{\partial \omega_i} \\ &= \frac{\partial (\sum_{i=1}^d \omega_i E[y_{i,t} | y_{m,t} \leq VaR_q^m])}{\partial \omega_i} \end{aligned} \quad (55)$$

which results in the expression from equation (54).  $ES_q^m$  denotes the expected shortfall for the entire financial system which can be constituted as sum of the weighted individual expected shortfall estimates.

Löffler and Raupach (2016) offer an analytical approach on the consistency and robustness of various newly trending return-based systemic risk measures such as the introduced CoVaR, Exposure  $\Delta$ CoVaR and MES. According to a linear factor model, Löffler and Raupach (2016) find that the Exposure  $\Delta$ CoVaR and the MES become substitutes within a linear model of normally distributed random variables and only differ by a usually small constant. They continue with an analysis of the direct effects of size, systemic and idiosyncratic risk on the risk measures and show some caveats which might raise incentives for further research and extensions on these risk measures.

Whereas the MES considers system returns to identify a crisis period, Oh and Patton (2017a) advance the measure of Brownlees and Engle (2016) by proposing the "kES" which measures the ES of a bank  $i$  conditional on a group of  $k$  banks being in distress:

$$kES_q^i = E[y_{i,t} | (\sum_{j=1}^d 1\{y_{j,t} < \mathbb{C}(y_{j,t})\}) > k] \quad (56)$$

In the analysis of Oh and Patton (2017a) the kES estimates rank higher than the overall MES. This measure can be of particular interest for periods where distress mainly oc-



curred at geographical distinct institutions among the SIFI banks. Although, the MES is doubted to hold as powerful tool for systemic risk measurement in practice among the financial industry and regulators, Idier et al. (2014) suggest to employ this measure especially on very large financial institutions. This work will focus on the MES and the Exposure  $\Delta$  CoVaR as extended systemic risk measures for individual institutions conditional on a distressed financial system.

## 5 Results

This section firstly will describe the underlying data. Then, results on the univariate properties of the data will be presented such as the distribution parameters. Next, dependence parameters calculated with an purely empirical and a conditional copula-based approach will be compared. This part will also include copula dependence parameters obtained by the numerical integration procedure. At the end, this work will focus on risk measure estimated in isolation or conditional on the systemic factor.

### 5.1 Data and univariate analysis

This systemic risk analysis focuses on a definition of systemic risk inherent in the systemically important financial institutions. The underlying data contains the daily return series of 28 financial institutions over the time range between 01 January 2007 and 31 December 2015. Only commercial banks that are declared as systemically important financial institutions by the Financial Stability Board in Basel are considered within the data set. Some Institutions, though declared as SIFI bank, are not included in the data since they either dropped out or joined the status of a SIFI bank during the observed time interval. The remaining 28 daily return series are constructed into a portfolio weighted according to each bank's size which is measured here as the market capitalization. The resulting portfolio is then the representation of the financial system. All banks and their corresponding characteristics such as their average size included in this analysis are shown in table 7. Among the observed SIFI banks, there are eight banks from the USA, four banks from the United Kingdom, three Chinese and Japanese, respectively and the ten largest European banks. While the SIFI status has been assigned to these 28 banks, three banks lost this status within the sample period due to declining systemic importance: Dexia, Commerzbank and Lloyds.

While the application of market return data for risk modeling in copula models is the benchmark in current and up-to-now research, the Credit Default Swap (CDS) market has grown sharply over the last two decades and thus enhances the practical capability of CDS spread data for quantitative research. Creal and Tsay (2015) use CDS spreads and stock returns to model a stochastic factor copula model. Data on CDS spreads were also employed by Oh and Patton (2017b) and Christoffersen et al. (2013) for dynamic copula models to assess system risk and credit risk, respectively. Due to the higher practical applicability of CDS spreads, the research on systemic risk and default probabilities has been shifting attention to CDS spread data (Dittmar, 2010). However, equity returns are still in highly frequent use for the mentioned research fields, such as in Oh and Patton

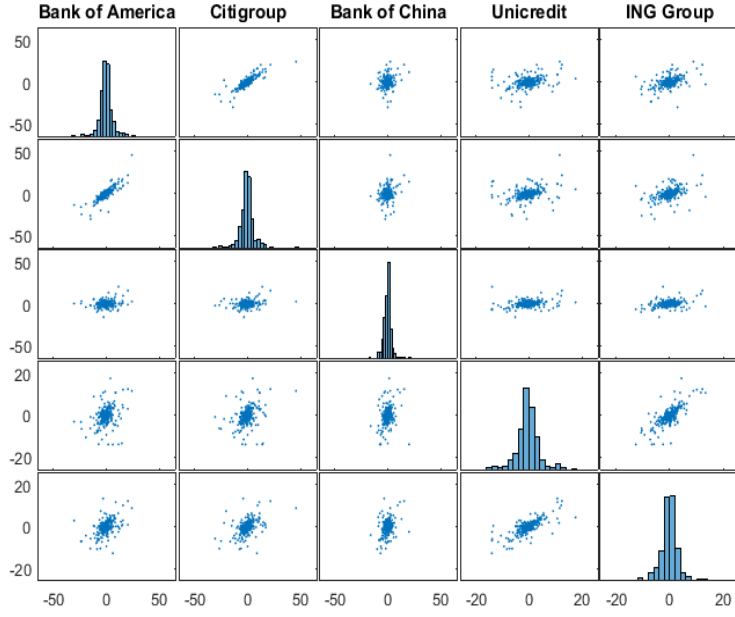


Figure 1: Density and scatter plots of the 2008 daily returns of Bank of America, Citigroup, Bank of China, Unicredit and ING Group

(2017a), Chen et al. (2017) and Acharya et al. (2017). He and Gong (2009) present a theoretical background to the application of equity prices on risk analyses. Based on the definition for the value of a company, the default event is defined as a company's disability to pay its debt rate with its equity returns.

To motivate an analysis on dependence measures, figure 1 plots the daily return series from the crisis period 2008 for two US American banks, that are the Bank of America and Citigroup and two European banks, that are Unicredit and the ING Group and, for the sake of comparability, the Bank of China. The plot clearly shows the geographical difference in the empirical dependence structures between the five banks. The scatter plots between the Bank of America and Citigroup strongly shows high empirical dependence in the tails. Same holds for the both European banks. While the Bank of China barely expresses any stronger dependence patterns with the US and European banks, the scatter plot with Unicredit fluctuates on higher level than with the both US banks. Surely, the plot also indicates dependence between the US and European banks. In general, the plotted univariate densities exhibit a leptokurtic shape suggesting heavy tails.

From figure 1 one can derive that the return series are not normally distributed in many years which is usual for financial data. After cleaning for non-zero means in the time series, the data are filtered by a GARCH process (cp. equation (19)) to control for lags in the conditional mean and volatility. Then, obtaining the *iid* standardized residuals for each SIFI, their marginal parameters can be estimated on basis of the *Skew t* distribution

with density from equation (22). The estimated skewness parameters for the standardized residuals are displayed in table 1 which shows skewness parameters close to zero on average.

Index (SIFI)	2007	2008	2009	2010	2011	2012	2013	2014	2015
1(JPM)	3.784	7.288	5.925	9.130	7.744	4.519	7.025	4.481	13.017
2(BAC)	3.623	5.220	4.679	6.416	6.261	11.851	4.831	6.470	8.066
3(BKM)	4.591	4.973	4.953	4.328	7.187	7.246	6.387	5.970	4.869
4(CITI)	3.572	6.791	4.879	8.996	5.165	8.813	12.028	5.144	5.731
5(GS)	5.482	4.473	4.940	4.547	5.688	5.908	8.828	26.640	9.388
6(MS)	4.453	4.076	4.120	18.297	5.528	12.373	5.639	45.875	6.911
7(SST)	4.309	5.023	4.076	6.919	5.209	3.883	3.964	5.603	4.934
8(WFC)	3.578	4.089	5.293	6.658	6.800	4.994	7.627	17.341	7.236
9(RBS)	9.353	3.771	5.657	8.036	8.556	9.705	6.252	4.731	8.512
10(BAR)	15.458	6.154	4.723	15.857	8.453	4.062	6.074	4.069	5.850
11(HSBC)	8.654	4.074	6.850	4.373	8.035	4.039	6.245	16.816	5.361
12(STAN)	9.718	4.604	5.668	8.276	6.705	5.514	5.773	4.282	4.394
13(BOC)	4.589	5.267	7.468	25.035	4.486	7.677	4.724	4.357	4.469
14(ICBC)	4.724	5.789	10.044	12.163	6.490	8.280	3.977	3.959	4.465
15(CCB)	12.007	7.916	21.386	11.611	9.003	15.207	3.801	5.356	4.926
16(BNP)	5.959	5.247	6.910	5.942	14.951	5.474	8.609	14.185	12.855
17(ACA)	10.487	5.591	6.346	6.790	8.224	10.121	9.341	9.336	4.664
18(GLE)	8.826	5.211	8.836	5.782	10.479	8.308	7.328	11.375	6.261
19(DB)	4.122	5.692	4.970	7.748	8.286	7.493	18.149	9.694	6.130
20(UCG)	6.029	5.652	33.437	5.352	28.937	5.213	8.566	10.969	6.407
21(ING)	299.770	6.273	7.213	6.788	22.105	10.250	10.927	7.960	15.898
22(SAN)	9.721	6.838	8.182	4.796	7.772	8.212	8.780	9.719	6.076
23(NDA)	6.282	5.112	12.327	6.996	5.880	7.463	10.425	4.430	4.743
24(CS)	5.148	6.538	66.054	4.950	5.515	5.144	5.017	11.448	3.810
25(UBS)	8.276	6.795	18.033	5.010	9.375	4.692	4.576	3.984	5.598
26(MTU)	10.684	12.367	4.088	7.039	7.010	50.139	4.660	9.960	5.524
27(MFG)	4.094	11.991	4.181	4.564	5.224	20.987	3.893	4.915	3.650
28(SMFG)	4.276	12.584	4.088	4.682	6.784	44.558	5.401	5.260	4.689
Average $\nu_i$	17.199	6.264	10.190	8.110	8.638	10.790	7.102	9.797	6.587
System $\nu$	10.802	4.866	7.543	4.426	19.080	9.218	13.332	42.925	6.189
Average $\lambda_i$	-0.013	0.048	-0.017	0.001	-0.027	0.044	-0.010	-0.032	-0.034
$\lambda_{sys}$	-0.219	0.001	-0.134	0.001	-0.103	-0.002	-0.180	-0.128	-0.070

Note: This table presents the degree of freedom parameters for each SIFI from 2007 to 2015, the corresponding averages and skewness parameter averages and both marginal parameters for the systemic factor.

Table 1: Marginal Parameters

In contrast to the idiosyncratic residuals, the system returns show skewness supporting the *Skew t* distributional assumption for the later simulated returns. Furthermore, the degrees of freedom for all SIFI banks and the systemic factor are shown in table 1. For most banks the degrees of freedom during the observed periods are fluctuating on relatively low level indicating heavy-tailed distributions. While the year 2012 contains relatively higher levels of degrees of freedom, such as for the Japanese banks (38.56 on average) and Morgan Stanley (12.37), those historical return series are probably exposed

to lighter tails than other banks' returns. Similar pattern can be detected for 2014 when Goldman Sachs (26.64), Morgan Stanley (45.88) and Barclays (16.82) show relatively lighter tail behavior based on their historical returns. During the periods from 2007 until 2009 including the financial crisis, the average degrees of freedom for US banks fluctuate around five suggesting heavy tails. This level is also reached by British and European SIFI banks during 2008 and by the Japanese banks in 2009. After years of slightly higher degrees, the levels fell again, partially sharply, in 2015. However, analyzing the marginal parameters of historical return series holds only as a description of the bank-specific characteristics but cannot offer significant insights on the actual dependence of SIFI banks on the system.

The next section will focus on dependence approaches measuring firstly, the dependence structure between the banks with empirical tail dependence and copula-implied tail dependence coefficients and secondly, the dependence between the SIFI banks and the financial system.

## 5.2 Results on dependence measures

The first part of this section deals with empirical tail dependence coefficients which measure dependence structures among the SIFI banks for extreme events. The second measure of this section is the copula-implied tail dependence. Both measures have been explained in section 2.2. Thirdly, the factor copula parameters from equation (14) obtained through the numerical integration on the maximum likelihood estimation (48) are presented.

Referring to the large variation of the marginal parameters in table 1, the marginal distributions of the SIFIs seem to be exposed to a certain heterogeneity in their distributional shapes, motivating the analysis to shift the focus to a non-parametric approach which circumvents the danger of a misspecified distributional assumption. That is why, this work now observes the non-parametric dependence measure of Schmidt and Stadtmüller (2006), that is the empirical lower tail dependence coefficient from equation (11). It measures the strength of the bivariate empirical dependence in the tails between two SIFIs on the interval  $[0, 1]$ . The results for all nine years are plotted in figure 5. While the empirical lower tail dependence coefficients increased from 2007 to 2008, the dependence reduces considerably in the after crisis periods. That goes hand in hand with the development at the interbank market after the fall of Lehman Brothers when banks hugely pulled down connections to other banks to isolate from counterparty risk. In 2011 facing the sharpening of the European sovereign debt crisis, the tail dependence peaked again when especially the coefficients between the US and the European SIFIs rose sharply near 1. In general across all years of the sample, the tail dependence is subject to the geograph-

ical origin of the banks. Hence, the tail dependence within each of the five geographical clusters can be clearly distinguished from cross-geographical coefficients, though the coefficients for the US, UK and European banks vary permanently on a higher level than the dependence coefficients with the Asian banks. Except for the crisis period in 2008, the Chinese and Japanese SIFIs have significantly lower coefficients with the Western banks. Unexpectedly, after three years of lower dependence, the tail dependence across all regions increased sharply in 2015 and the Japanese banks yield higher tail dependence coefficients with all other banks than for any sample period prior.

The focus now lies on the copula-implied lower tail dependence from equation (5) being defined in the interval  $[0, 1]$  which measures the dependence in the lower left quadrant of a bivariate distribution. This approach is based on the copula dependence parameters of the one-factor model  $\alpha_i$ . Whereas Chen and Nasekin (2017) analyze the copula-implied dependence coefficients of non-central banks on the central institution for each year, Oh and Patton (2017a) apply the approach presented in section 2.2 which considers the dependence parameters for each SIFI on the systemic factor. Then, using the estimated marginal parameters from table 1 and according to the *Skew  $t - t$*  distributional assumption for the conditional copula in (18), the degrees of freedom of the systemic factor for each year are set equal for  $F_Z$  and  $F_\varepsilon$ . Hence, this setting guarantees equal tail indices for  $Z$  and  $\varepsilon$  which avoids boundary results but enables to calculate the copula-implied tail dependence coefficient by the procedure shown in section 2.2. The resulting copula-implied lower tail dependence matrices for each year are given in figure 6.

Figure 6 clearly shows an highly interconnected financial system in 2007 which became pertubated in 2008 with sharply increasing tail dependence within the UK and European banking sector with average coefficient within the European banking sector of around 0.92. While in 2009 the tail dependence in Europe decreased essentially, the US banks' tail dependencies increased again until 2010/2011 when contemporarily the European banks peaked at maximal lower tail dependence coefficients for BNP Paribas (0.945) and Societe Generale (0.949). This development is probably triggered by the European debt crisis and while the crisis slowed down over the periods from 2012 to 2014, almost all lower tail dependence coefficients decreased. Finally in 2014, all regions reached lowest clustered coefficients in 2014 over the entire observation period with an average tail dependence for USA (0.61), UK (0.58), China (0.169), Europe (0.813) and Japan (0.167) conditional on the respective region of origin. Whereas the overall tail dependence in 2014 was 0.451, it sharply increased in 2015 up to 0.639 being higher than in 2008 (0.624). The origin-clustered tail dependence coefficients rose heavily compared to the previous years

for USA (0.758), UK (0.801), China (0.461), Europe (0.880) and Japan (0.357).<sup>3</sup> Again, as in figure 5, the Asian banks are subject to significantly lower tail dependence coefficients which, at an overall perspective, show the relative delimitation of the Asian from the Western banking sector and potentially suggest the Asian banks to be more stable against system-wide shocks. Still, it is to take into account that the coefficients for the Asian banks increased sharply in 2015 which, together with the results from figure 5, implies an higher interconnectedness between the SIFI banks on a worldwide level.

The main results from figure 5 and 6 are consistent with the findings of Chen et al. (2017) and Chen and Nasekin (2017) who analyze empirical and copula-implied tail dependence coefficients in a network-based factor copula model approach. In Chen and Nasekin (2017) the copula-implied tail dependence is investigated conditional on central nodes that represent those SIFIs which hold a larger potential for an overall tail risk. The obtained copula-implied tail dependence matrices rank on slightly lower range than the results from Oh and Patton (2017a) who calculated the lower (0.94) and upper (0.09) tail dependence coefficients for the S&P100 index from April 2008 until December 2010 containing 18 financial industry companies. Since their systemic risk analysis concentrates on the S&P100 index which only considers US companies, their setting completely isolates geographical clustering which together with the different observation period and different common factor is responsible for the higher lower tail dependence coefficient for the US financial industry.

Now, this section will analyze the copula parameters estimated by the log-likelihood function (48) on the factor copula density (46) which has to be solved numerically according to section 3.2. After obtaining the results from the numerical integration and optimization, the factor copula is simulated first which is applied on the inverse *Skew  $t - t$*  distribution using the estimated degree of freedom and skewness parameters and rescaled with the mean effect and GARCH effect from the GARCH model (19). The resulting simulated returns are now used for the further analysis. A copula dependence parameter  $\alpha_{i|j}$  describes the dependence between the returns  $Z_i$  on  $Z_j$ . The results for this part are estimated for twofold directions. On the one hand, the copula dependence parameters are estimated for the system returns conditional on the each SIFI bank. On the other hand, the other direction is considered, i.e. the copula dependence parameters conditioning the simulated SIFI returns on the system returns according to equation (14) are obtained in a simultaneous estimation.

The first setting analyzes the financial system exposure to each SIFI bank which directly

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<sup>3</sup>Note that the color scale at the right-side of each year's plot in figure 6 varies and hence the figures are to interpret conditional on the different scale interval.

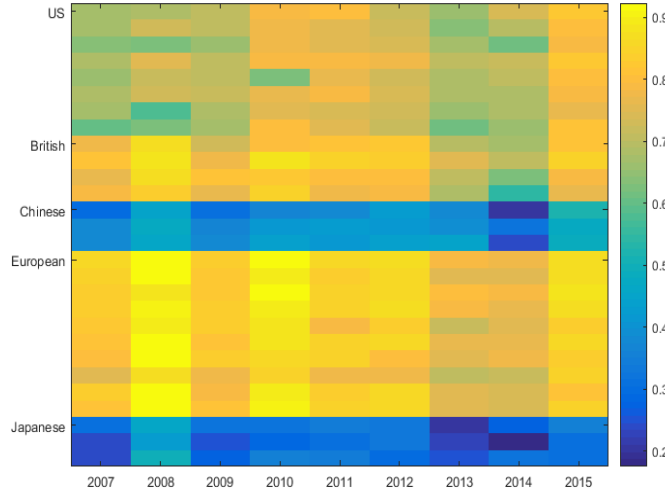


Figure 2: Copula Dependence parameters for the system returns conditional on the SIFI returns from 2007 until 2015

addresses to the *too-big-to-fail* topic. The results for nine years are plotted in figure 2. Across all institutions, the dependence parameter for the financial system increased in 2008 and also varies on high level during the European sovereign debt crisis in 2010 and 2011 expressing higher systemic risk held by each institution compared to quieter years. While the sensitivity of the financial system on the SIFI banks decreases continuously from 2012 to 2014, it again sharply peaks in 2015 pointing to an higher systemic importance of each SIFI bank in the most recently observed period.

Whereas the average factor copula parameter in 2014 was around 0.584, it reached 0.733 in 2015 which is approximately as high as in 2008 (0.731) showing that the financial system again reached a dependence structure that is similar to the dependence in the crisis period. However, as there was no severe system-wide financial crisis in 2015, the dependence structure might have been even stronger if 2015 was a crisis period. Across the entire observation periods, the system was most vulnerable to European banks and in particular, to BNP Paribas (0.901 on average), Credit Agricole (0.878), Societe Generale (0.903), Deutsche Bank (0.872) and the ING Group (0.871). During the quieter period from 2012 to 2014 all European banks lowered their dependencies from 0.88 down to 0.80, except for Santander which continued growing from 0.81 up to 0.86.<sup>4</sup>

While US banks represented especially high systemic risk in 2010 and 2015 with values in the latter period for JP Morgan (0.781) and Citigroup (0.793), British banks peaked in 2008, 2010 and 2015 when Barclays obtained values such as 0.905, 0.898 and 0.862, re-

<sup>4</sup>This development of Santander, for instance, is not displayed by figure 3 in which Santander reduces its dependence instead.



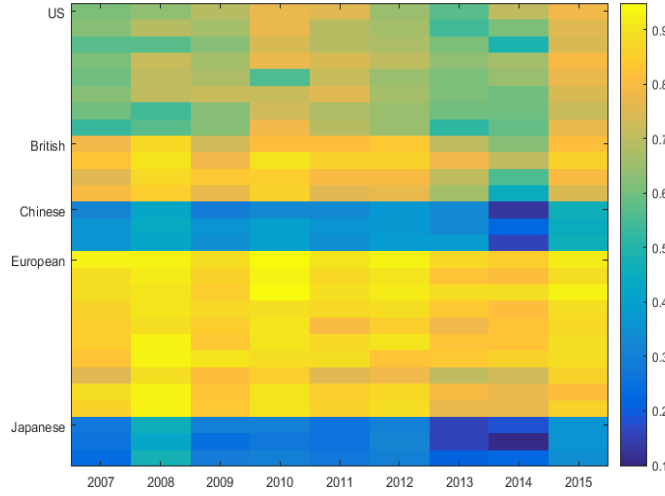


Figure 3: Copula Dependence parameters for the SIFI returns conditional on the system returns from 2007 until 2015

spectively. Even though, the Asian bank range on average between 0.3 and 0.4, the sharp increase in 2015 additionally stresses the risen systemic risk associated by all SIFIs. While both, Chinese and Japanese banks obtained values around 0.17 in 2014, the parameter values peaked in 2015 at around 0.45 and 0.36, respectively. These results in total imply an increased systemic importance among all SIFI banks in the most recent period.

The analysis now turns around the exposure direction and investigates the dependence of each SIFI bank on the financial system which is rather referred to the *too-interconnected-to-fail* topic. The corresponding results for nine years are plotted in figure 3. Supposing that the Lehman Brothers shock rendered in a systemic crisis, this factor copula parameter matrix shows how severely the SIFI banks would be exposed to a systemic event. Again 2007 and 2008, shows risen factor copula parameters among all banks. While the parameters from figure 2 and 3 differ in level and changes from year to year, the overall pattern does not change significantly.<sup>5</sup> Certainly, some values might differ in level and periodwise change but still, it holds that the years 2008, 2010, 2011 and 2015 yield the highest factor copula parameters. In contrast to figure 2 the absolute difference between the parameters for the US and European banks is significantly lower (by approximately 0.1) implying that the vulnerability of US banks to the financial system is relatively higher than vice versa. The opposite holds for the European banks which would potentially affect the financial system more severely than the financial system would affect them. Credit Suisse however, reached 0.923 in 2008, the highest value that period among all banks but afterwards continuously reduced its dependence on the system and while the dependen-

<sup>5</sup>Note again, that the color bar at the right-hand side of the plots is differently scaled.

cies of the other European banks varies between 0.86 and 0.933 in 2015, Credit Suisse obtained a much lower level of 0.80. BNP Paribas and Societe Generale, each provide three times the highest overall dependence parameter. These results are supported by Chen and Nasekin (2017) who found BNP, Credit Agricole and Societe Generale to be exposed to high tail dependence even when conditioning on a non-central node. Among the US banks, there is a high variation in the exposures to the system. That is, Bank of America in 2007, Morgan Stanley in 2009, Wells Fargo in 2010, JP Morgan in 2011, and 2014, Citigroup in 2008, 2012, 2013 and 2015 yielded the highest values among the US banks with extrema at around 0.83 in 2015. Among the British banks, Barclays represented again the highest dependence on the system, though HSBC and Royal Bank of Scotland yield similarly high values (on average around 0.80 for the three mentioned). For most years, the British and European banks barely differ much in terms of level. Apart from the Western SIFIs, the Chinese banks again rank on profoundly lower levels but with dependence on the systemic factor in 2015 (0.50 on average) ranging higher than in 2008. Japanese banks instead only fluctuate around their overall average values (around 0.3) in 2015 whereas their systemic importance in figure 2 rose beyond their average. At the end, these results show an increasing sensitivity of all SIFIs to the systemic factor implying that the financial system reached a higher interconnectedness making the SIFIs more affected by a potential systemic extreme event.

### 5.3 Results on systemic risk measures

This analysis will present the results for the risk measures introduced in section 4 obtained from the simulated return series. As the focus of this work mainly lies on the potential impact of the financial system on SIFI banks, the systemic risk is measured by the vulnerability of each institution to the system.

Given the factor copula model from equation (14) the VaR and the ES are calculated from the simulated returns for each SIFI and displayed in table 3 and 4. Since these isolated risk measures do not provide much power for identifying systemic risk in financial institutions, this section concentrates on the conditional risk measures which impose a stressed event on the systemic factor.

Table 3 lists the VaR estimates of all SIFIs from 2007 until 2015. In line with results from empirical research, 2008 shows an especially severe condition to the US banks, such that Citigroup and Morgan Stanley with -10.3% and -9.1% at 95% VaR. During the sovereign debt crisis in Europe in 2011 particularly BNP Paribas, Credit Agricole, Societe Generale, Unicredit and ING Group suffered according to their VaR estimates which ranked as the largest in absolute terms from 2010 until 2012. While the VaR in 2013 and 2014 moved

down to low pre-crisis level of 2007, the VaR estimates sharpened in 2015 with British and European banks yielding the highest values in absolute terms.

As expected, the corresponding Expected Shortfall estimates show a similar pattern with particularly severe periods for British banks, i.e. Barclays leading the list in 2008 and 2010, the Royal Bank of Scotland yielding the largest negative values in 2009 and 2013 and Standard Chartered in 2015. While Standard Chartered's ES estimates used to vary below the average ES, it almost doubled from 2014 to 2015. While BNP, Credit Agricole and Societe Generale had, together with Unicredit, the absolute highest ES values in 2008, those three banks outperformed Deutsche Bank, Unicredit, ING and Santander as well as Standard Chartered and Bank of China in 2015. Most of the banks obtained higher ES estimates in 2015 than in 2013 and 2014.

The next measure for systemic risk, the Exposure  $\Delta\text{CoVaR}$  (ExCoVaR) becomes more and more applied in research on systemic risk such as in Pagano and Sedunov (2014). This measure gives an intuition on how much a single institution is affected by a system downturn as stated by equation 53. The VaR and ES for 2008 emphasized the severely adverse returns of the US banks in particular, the ExCoVaR though quantifies the impact of a market downturn on the individual VaR levels. Table 5 gives the individual exposures on an adverse market move from median state to market returns conditional on their 95% and 99% VaR estimates. Although, 2008 is characterized as the most severe crisis year by highest losses given VaR and ES, the ExCoVaR obtains its maximum value in 2009 for the Royal Bank of Scotland (-8.477). The case of RBS in 2009 obviously shows the different nature of the ExCoVaR compared to the VaR and ES. On the one hand, 2008 generated the highest total losses, on the other hand, the ExCoVaR implies that an adverse market movement would have hit a few banks more severely in 2009 than in 2008. This relation can be stated clearer when drawing a comparison between the differences in the averages of the 2008 and 2009 VaR and ExCoVaR. While the average 2008 and 2009 95% VaR yields -5.76 and -3.66 respectively, the corresponding averages for the ExCoVaR are -5.04 and -4.16 implying that the SIFI banks would have suffered from an equivalent systemic shock in 2009 more harmful than one could deduce from the VaR and ES values. Those discoveries underline the significance of risk measures conditioning one market participant on a severe event of another market participant.

Similar to the ExCoVaR, the Marginal Expected Shortfall measures the institution-specific exposure to the distressed financial system. Figure 4 illustrates the MES estimates from table 6 for nine years with each year having two values, the 95% quantile listed first, the 99% quantile second. As described in section 4 the MES also holds as measure for the risk contribution of bank  $i$ 's idiosyncratic risk to the financial system's overall risk

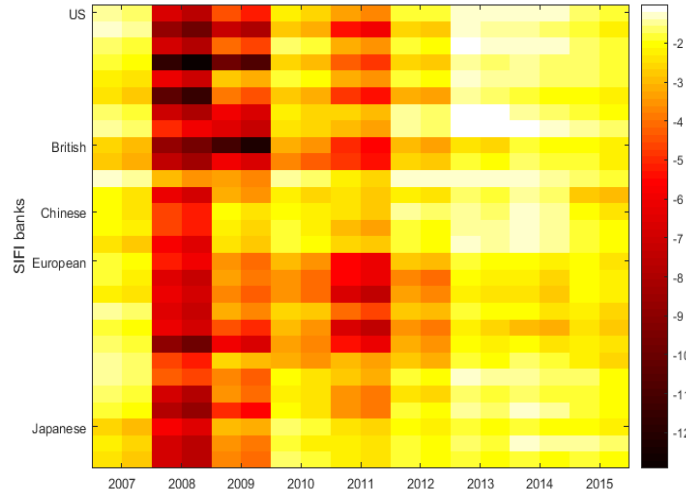


Figure 4: Marginal Expected Shortfall at 95% and 99% quantiles for 2007-2015

(Acharya et al., 2017). Unsurprisingly the banks ranked highest at the ExCoVaR are also skyrocketing the MES estimates, though it should be noticed that the risk contribution of different institutions to the overall systemic risk varies from the findings of the ExCoVaR. Hence, the MES relatively stresses the systemic sensitivity on JP Morgan's, Wells Fargo's and HSBC's idiosyncratic risks in 2008 compared to the previous risk measures. That is, setting the MES into relation to the ExCoVaR, it shows that those banks' risk taking contribute to the overall systemic risk relatively stronger than implied by the ExCoVaR. For the year 2009 the MES distributes relatively higher sensitivity to Goldman Sachs, the Bank of China and the Deutsche Bank in comparison to the ExCoVaR.

The analysis of Acharya et al. (2017) concentrates on the predictability of systemic risk by conditional risk measures such as the ES and the MES. Thereby, the MES provides significant predictive power for those companies with the most extreme tail events during the financial crisis from 2007 until 2009 found within the frames of an OLS regression and a Tobit analysis. In the comparative analysis of Benoit et al. (2013) the MES outperforms the  $\Delta\text{CoVaR}$  by assessing their power of identifying an increase in systemic risk at the example of Lehman Brothers.

## 6 Conclusion

This work investigates systemic risk in financial institutions for the most systemically important banks from pre-crisis period 2007 until 2015. Firstly, this analysis focuses on the factor copula parameters which provide an adequate estimation for the individual institution's dependence on the systemic factor. Subsequently, further dependence measures are proposed based on the factor-copula model but also estimated by a purely empirical approach. The following derivation of the factor-copula model yields a copula density function which on the one hand, allows for flexible constructions with different distributional choices and on the other hand, provides a suitable framework for high-dimensional estimation. The latter comes from the dimension reduction property of the factor model. This framework considers a *Skew  $t - t$*  one-factor copula model which produces asymmetric and tail dependence that achieves a better fit to the financial data. Numerical methods are then applied to solve the maximum likelihood estimation in order to simulate the return series on which basis the estimation of the risk measures takes place. At this final stage, this analysis focuses on the systemic risk measure of the Exposure  $\Delta\text{CoVaR}$  of Adrian and Brunnermeier (2008) and the MES of Acharya et al. (2017) to examine the exposure of the financial institution to a system-wide shock of the financial sector.

According to the factor copula parameters, they illustrate higher dependence structures during crisis periods around 2008 and 2011 whereas the dependence decreases essentially after the European debt crisis prior an renewed increase in 2015. Since 2015 is not subject to an adverse economic development compared to the prior years, the estimates imply that the interconnectedness within the financial sector increased without moving into a tail event. This is backed by estimates which are based on the factor copula (Oh and Patton, 2017a) but also by the nonparametric empirical tail dependence coefficients of Schmidt and Stadtmüller (2006). Deviations between the empirical tail dependence and the copula-implied tail dependence coefficients might be due to variation in the degrees of freedom and the skewness parameter which is circumvented by the nonparametric empirical measure.

When measuring systemic risk the ExCoVaR and the MES are to prefer over the isolated measures VaR and ES by financial institutions and regulators according to their robustness and possible predictive power (Idier et al., 2014). While the VaR and the ES assign higher values to the American banks in 2008, the MES shows an relatively higher exposure of British and European banks compared to the corresponding VaR and ES estimates. This being said, the ExCoVaR and the MES prove their importance by assessing systemic risk instead of individual maximal loss estimation of VaR and ES. The conditional risk

measures clearly show the relative high overall exposure of the European SIFIs across all sample periods whereas the US and British banks obtained the highest values during the financial crisis periods. The topic of *too-big-to-fail* is connected to the nature of the MES, which is driven by larger bank sizes that might be responsible for the relatively high systemic risk measures of the Asian institutions in relation to their comparably low dependence coefficients.

Since all measures estimated within this work agree to an increase of the overall systemic risk in 2015, this analysis should become extended by more recent data. The higher dependence measured by the factor copula parameters and the tail dependencies directly addresses to the topic of *too-interconnected-to-fail*. Subsequent research might apply multi-factor copula models to identify heterogeneous dependence with different drivers for systemic instability and individual vulnerability. Future research may also connect to the various and frequently trending analysis on risk measure extensions to study the capability of systemic risk measures, such as the MES, to provide appropriate predictions on systemic tail events.

Apart from the methodology itself, the majority of systemic risk analyses base their estimations on equity and CDS data while option price data may offer opportunities for more reliable predictive modeling of systemic risk measures.

## 7 Figures and Tables

Index	SIFI	Firm Size	Debt Ratio	Bucket	Country
1	JP MORGAN CHASE	21.506	0.261	4	U.S.
2	BANK OF AMERICA	21.446	0.302	2	U.S.
3	BANK OF NEW YORK MELLON	19.499	0.095	1	U.S.
4	CITIGROUP	21.359	0.300	3	U.S.
5	GOLDMAN SACHS	20.624	0.509	2	U.S.
6	MORGAN STANLEY	20.501	0.417	2	U.S.
7	STATE STREET	19.106	0.153	1	U.S.
8	WELLS FARGO	20.980	0.183	1	U.S.
9	ROYAL BANK OF SCTL	21.588	0.252	1	U.K.
10	BARCLAYS	21.604	0.286	3	U.K.
11	HSBC	21.682	0.127	4	U.K.
12	STANDARD CHARTERED	20.136	0.187	1	U.K.
13	BANK OF CHINA	21.200	0.160	1	China
14	ICBC	21.508	0.089	1	China
15	CHINA CON.BANK	21.281	0.092	1	China
16	BNP PARIBAS	21.684	0.136	3	France
17	CREDIT AGRICOLE	21.489	0.211	1	France
18	SOCIETE GENERALE	21.184	0.139	1	France
19	DEUTSCHE BANK	21.630	0.200	3	Germany
20	UNICREDIT	20.929	0.360	1	Italy
21	ING GROEP	21.156	0.103	1	Netherlands
22	SANTANDER	21.158	0.368	1	Spain
23	NORDEA BANK	20.476	0.326	1	Sweden
24	CREDIT SUISSE GROUP N	20.744	0.339	2	Switzerland
25	UBS GROUP	21.008	0.251	1	Switzerland
26	MITSUBISHI UFJ	21.533	0.159	2	Japan
27	MIZUHO	21.247	0.233	1	Japan
28	SUMITOMO.MITSUI	21.044	0.125	1	Japan

Note: The Debt ratio gives the ratio of total debt to total asset, and the bank size, as log value of total assets denominated in the USD, are shown as their mean value during time period (2007-2015). The buckets assigned by BCBS correspond to required level of additional common equity loss absorbency as a percentage of risk-weighted assets from 3.5% (Bucket 5), 2.5%(Bucket 4), 2.0%(Bucket 3), 1.5%(Bucket 2) to 1%(Bucket 1).

Table 2: Summary information on systemically important financial institutions

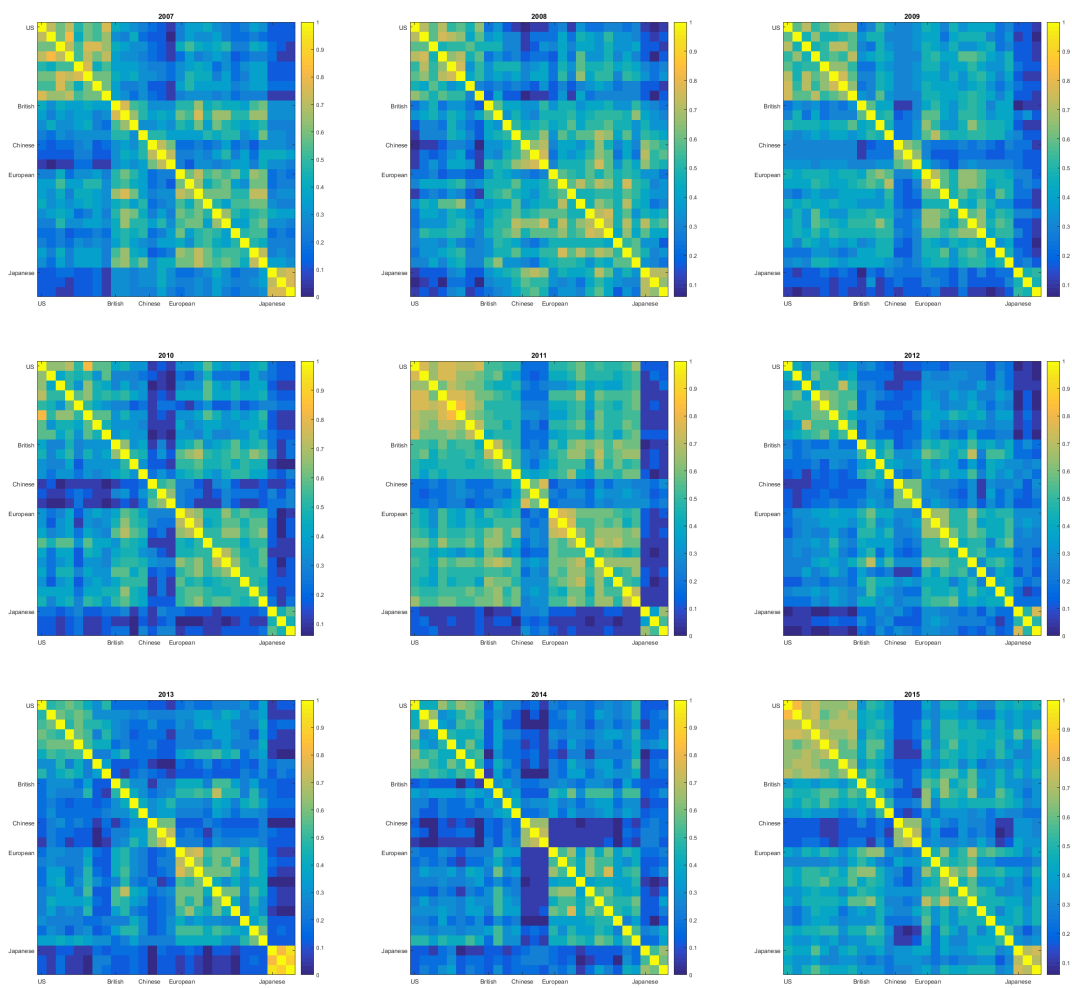


Figure 5: Empirical Tail Dependence coefficients for the SIFIs for 2007-2015



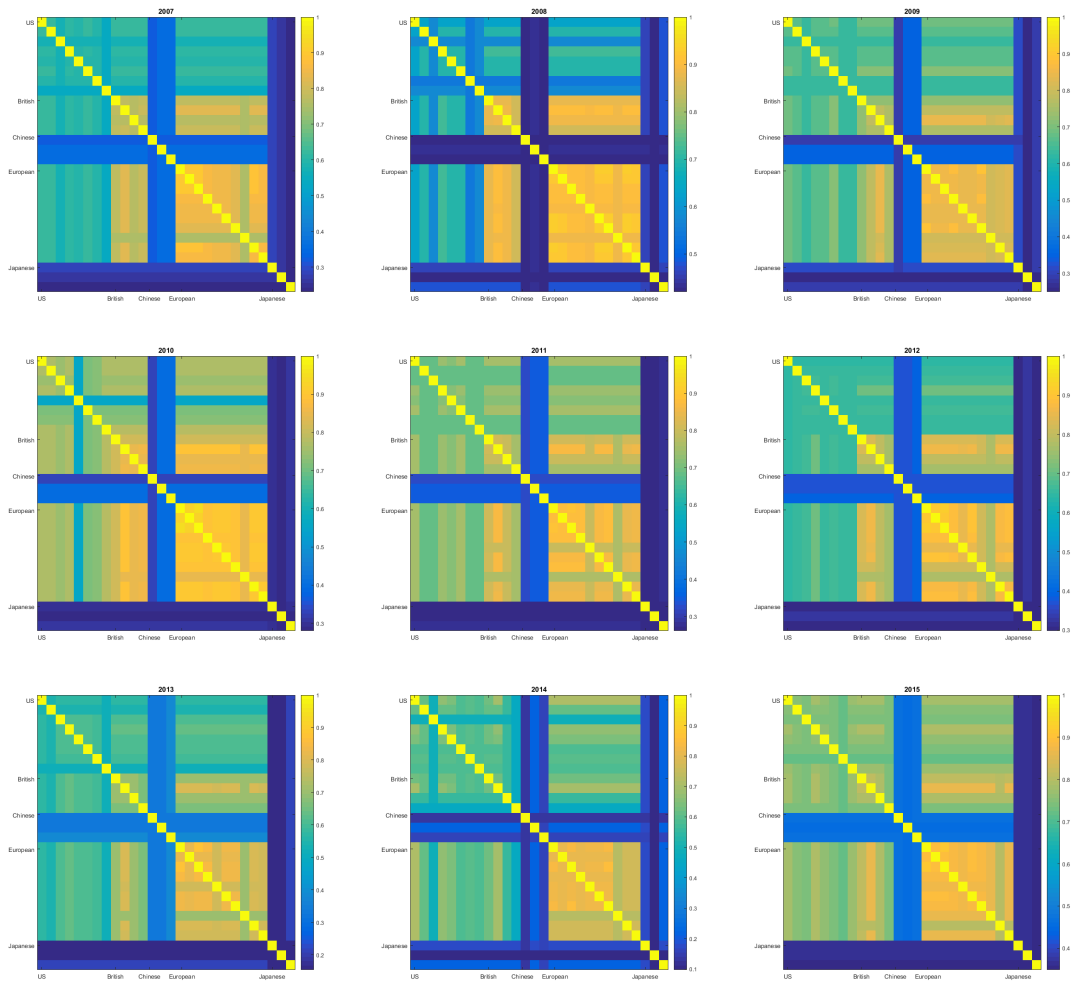


Figure 6: Copula-implied Tail Dependence coefficients for the SIFIs for 2007-2015

Index (SIFI)	2007		2008		2009		2010		2011		2012		2013		2014		2015	
	$VaR_{95}$	$VaR_{99}$	$VaR_{95}$	$VaR_{99}$	$VaR_{95}$	$VaR_{99}$	$VaR_{95}$	$VaR_{99}$	$VaR_{95}$	$VaR_{99}$	$VaR_{95}$	$VaR_{99}$	$VaR_{95}$	$VaR_{99}$	$VaR_{95}$	$VaR_{99}$	$VaR_{95}$	$VaR_{99}$
1(JPM)	-1.330	-1.659	-5.611	-7.105	-3.647	-4.894	-1.938	-2.534	-2.800	-3.508	-1.437	-1.800	-1.013	-1.310	-0.929	-1.298	-1.290	-1.819
2(BAC)	-1.137	-1.402	-7.704	-9.349	-5.971	-7.762	-2.293	-2.957	-4.700	-5.684	-2.064	-2.671	-1.089	-1.409	-1.184	-1.657	-1.489	-2.074
3(BKM)	-1.359	-1.726	-5.917	-7.363	-3.425	-4.434	-1.398	-1.839	-2.808	-3.483	-1.509	-1.889	-0.992	-1.285	-0.930	-1.312	-1.247	-1.751
4(CITI)	-1.642	-1.983	-10.307	-12.301	-8.537	-10.451	-2.112	-2.844	-3.729	-4.577	-2.069	-2.609	-1.296	-1.669	-1.128	-1.564	-1.301	-1.817
5(GS)	-1.849	-2.330	-5.393	-6.604	-2.231	-3.090	-1.555	-2.036	-2.767	-3.395	-1.514	-1.908	-1.140	-1.479	-1.118	-1.565	-1.337	-1.857
6(MS)	-2.119	-2.617	-9.106	-11.011	-3.153	-4.245	-2.297	-2.975	-4.233	-5.198	-2.501	-3.122	-1.274	-1.682	-1.356	-1.923	-1.526	-2.097
7(SST)	-1.453	-1.847	-6.145	-7.549	-4.960	-6.392	-1.925	-2.531	-2.294	-2.897	-1.193	-1.485	-0.851	-1.116	-1.129	-1.574	-1.392	-1.916
8(WFC)	-1.314	-1.631	-4.229	-5.491	-5.347	-6.916	-1.940	-2.563	-2.507	-3.169	-1.200	-1.497	-0.843	-1.091	-0.855	-1.208	-1.112	-1.555
9(RBS)	-2.341	-2.855	-7.847	-9.199	-9.530	-11.811	-2.571	-3.407	-4.383	-5.365	-2.390	-3.056	-2.041	-2.586	-1.385	-1.941	-1.564	-2.142
10(BAR)	-2.502	-3.059	-6.556	-7.935	-4.729	-6.360	-3.128	-4.042	-4.198	-5.204	-2.020	-2.563	-1.490	-1.891	-1.331	-1.821	-1.492	-2.069
11(HSBC)	-1.095	-1.365	-2.550	-3.234	-2.733	-3.596	-1.216	-1.581	-1.988	-2.482	-1.021	-1.284	-1.054	-1.335	-1.052	-1.445	-1.257	-1.734
12(STAN)	-1.764	-2.256	-5.250	-6.504	-2.433	-3.336	-1.855	-2.457	-2.132	-2.682	-1.769	-2.200	-1.414	-1.770	-1.161	-1.567	-1.291	-2.930
13(BOC)	-1.826	-2.276	-3.970	-4.929	-1.602	-2.245	-1.634	-2.146	-2.105	-2.621	-1.267	-1.580	-1.219	-1.545	-0.976	-1.389	-1.680	-2.290
14(ICBC)	-1.737	-2.208	-3.937	-4.970	-1.810	-2.466	-1.622	-2.117	-2.574	-3.230	-1.474	-1.835	-1.180	-1.487	-0.965	-1.356	-1.445	-1.978
15(CCB)	-2.124	-2.734	-4.845	-6.059	-1.977	-2.689	-1.520	-2.013	-2.208	-2.766	-1.545	-1.922	-1.175	-1.479	-1.065	-1.497	-1.455	-1.994
16(BNP)	-1.545	-1.923	-4.571	-5.588	-2.833	-3.835	-2.584	-3.334	-4.791	-5.930	-2.200	-2.800	-1.518	-1.956	-1.600	-2.186	-1.611	-2.284
17(ACA)	-1.678	-2.062	-5.691	-6.930	-2.745	-3.657	-3.017	-3.858	-4.908	-5.974	-3.010	-3.826	-1.717	-2.238	-1.713	-2.396	-1.529	-2.151
18(GLE)	-1.927	-2.375	-5.401	-6.587	-3.041	-4.043	-3.036	-3.907	-5.962	-7.235	-2.694	-3.494	-1.772	-2.303	-1.939	-2.631	-1.506	-2.159
19(DB)	-1.279	-1.592	-5.649	-6.828	-2.522	-3.482	-2.137	-2.760	-3.583	-4.470	-2.212	-2.773	-1.623	-2.050	-1.655	-2.232	-1.752	-2.432
20(UCG)	-1.569	-1.945	-5.419	-6.537	-3.556	-4.728	-2.614	-3.332	-5.874	-7.127	-2.859	-3.656	-1.922	-2.493	-2.257	-3.106	-1.833	-2.577
21(ING)	-1.473	-1.820	-7.793	-9.363	-4.936	-6.428	-2.733	-3.570	-4.651	-5.819	-2.615	-3.283	-1.703	-2.213	-1.646	-2.290	-1.674	-2.380
22(SAN)	-1.259	-1.583	-3.977	-4.918	-2.139	-2.899	-2.642	-3.367	-2.619	-3.291	-2.299	-2.883	-1.512	-1.935	-1.310	-1.840	-1.753	-2.397
23(NDA)	-1.213	-1.526	-3.601	-4.455	-2.991	-4.010	-1.731	-2.272	-2.475	-3.092	-1.522	-1.899	-1.131	-1.468	-1.087	-1.514	-1.328	-1.865
24(CS)	-1.376	-1.703	-6.027	-7.392	-2.854	-3.880	-1.880	-2.397	-3.067	-3.770	-1.922	-2.383	-1.365	-1.748	-1.259	-1.724	-1.376	-1.909
25(UBS)	-1.657	-2.033	-6.902	-8.351	-4.146	-5.334	-1.749	-2.276	-3.008	-3.750	-1.549	-1.950	-1.203	-1.538	-0.993	-1.369	-1.398	-1.985
26(MTU)	-2.308	-2.831	-4.940	-6.105	-2.455	-3.111	-1.405	-1.832	-2.050	-2.563	-1.687	-2.130	-1.542	-2.023	-1.464	-2.024	-1.536	-2.186
27(MFG)	-2.002	-2.437	-5.890	-7.250	-3.063	-3.801	-1.578	-2.045	-1.898	-2.353	-1.549	-1.963	-1.325	-1.729	-1.035	-1.416	-1.131	-1.615
28(SMFG)	-2.119	-2.595	-5.901	-7.223	-3.136	-3.918	-1.367	-1.795	-1.892	-2.365	-1.476	-1.868	-1.504	-2.003	-1.498	-2.037	-1.388	-1.962
Average	-1.678	-2.085	-5.755	-7.040	-3.661	-4.779	-2.053	-2.671	-3.293	-4.071	-1.877	-2.369	-1.354	-1.744	-1.286	-1.782	-1.485	-2.069
System	-1.645	-2.048	-5.192	-6.414	-3.387	-4.450	-1.918	-2.504	-2.853	-3.546	-1.681	-2.116	-1.248	-1.605	-1.153	-1.604	-1.446	-2.009

Table 3: Value at Risk at 95% and 99% quantiles for each SIFI

Index (SIFI)	2007		2008		2009		2010		2011		2012		2013		2014		2015	
	$ES_{95}$	$ES_{99}$	$ES_{95}$	$ES_{99}$	$ES_{95}$	$ES_{99}$	$ES_{95}$	$ES_{99}$	$ES_{95}$	$ES_{99}$	$ES_{95}$	$ES_{99}$	$ES_{95}$	$ES_{99}$	$ES_{95}$	$ES_{99}$	$ES_{95}$	$ES_{99}$
1(JPM)	-1.499	-1.724	-6.555	-7.430	-4.428	-5.094	-2.302	-2.577	-3.246	-3.634	-1.657	-1.875	-1.176	-1.324	-1.147	-1.345	-1.602	-1.888
2(BAC)	-1.273	-1.454	-8.743	-9.706	-7.093	-8.050	-2.699	-3.005	-5.320	-5.860	-2.432	-2.796	-1.264	-1.424	-1.463	-1.717	-1.834	-2.149
3(BKM)	-1.547	-1.799	-6.830	-7.677	-4.057	-4.596	-1.668	-1.871	-3.233	-3.604	-1.739	-1.967	-1.152	-1.299	-1.155	-1.360	-1.544	-1.816
4(CITI)	-1.817	-2.051	-11.567	-12.734	-9.736	-10.758	-2.559	-2.896	-4.264	-4.729	-2.396	-2.720	-1.500	-1.685	-1.385	-1.619	-1.605	-1.884
5(GS)	-2.096	-2.425	-6.158	-6.867	-2.769	-3.228	-1.849	-2.071	-3.163	-3.508	-1.753	-1.989	-1.326	-1.495	-1.382	-1.622	-1.644	-1.924
6(MS)	-2.375	-2.715	-10.309	-11.424	-3.837	-4.420	-2.711	-3.024	-4.842	-5.371	-2.877	-3.250	-1.498	-1.701	-1.691	-1.996	-1.863	-2.170
7(SST)	-1.655	-1.925	-7.032	-7.854	-5.857	-6.622	-2.295	-2.575	-2.674	-3.005	-1.370	-1.545	-0.996	-1.128	-1.392	-1.631	-1.701	-1.983
8(WFC)	-1.477	-1.694	-5.026	-5.765	-6.330	-7.168	-2.321	-2.608	-2.924	-3.287	-1.380	-1.557	-0.979	-1.102	-1.063	-1.253	-1.373	-1.612
9(RBS)	-2.604	-2.956	-8.701	-9.492	-10.959	-12.177	-3.082	-3.467	-5.003	-5.541	-2.794	-3.193	-2.340	-2.611	-1.713	-2.012	-1.905	-2.216
10(BAR)	-2.788	-3.169	-7.427	-8.234	-5.751	-6.622	-3.686	-4.108	-4.833	-5.385	-2.349	-2.675	-1.709	-1.909	-1.620	-1.883	-1.832	-2.144
11(HSBC)	-1.233	-1.418	-2.982	-3.382	-3.274	-3.735	-1.439	-1.607	-2.300	-2.570	-1.181	-1.339	-1.208	-1.348	-1.284	-1.495	-1.538	-1.795
12(STAN)	-2.016	-2.353	-6.042	-6.776	-2.999	-3.481	-2.223	-2.500	-2.479	-2.780	-2.030	-2.289	-1.609	-1.786	-1.401	-1.619	-2.627	-3.026
13(BOC)	-2.057	-2.365	-4.576	-5.137	-2.005	-2.348	-1.947	-2.183	-2.430	-2.713	-1.457	-1.645	-1.397	-1.560	-1.220	-1.442	-2.040	-2.369
14(ICBC)	-1.978	-2.301	-4.590	-5.194	-2.221	-2.572	-1.924	-2.152	-2.988	-3.347	-1.693	-1.910	-1.348	-1.500	-1.196	-1.405	-1.759	-2.047
15(CCB)	-2.437	-2.854	-5.612	-6.323	-2.423	-2.803	-1.821	-2.048	-2.560	-2.866	-1.774	-2.000	-1.342	-1.493	-1.320	-1.552	-1.773	-2.063
16(BNP)	-1.739	-1.998	-5.214	-5.809	-3.461	-3.996	-3.042	-3.388	-5.510	-6.134	-2.563	-2.923	-1.758	-1.976	-1.946	-2.261	-2.008	-2.371
17(ACA)	-1.875	-2.138	-6.474	-7.199	-3.317	-3.804	-3.530	-3.918	-5.581	-6.166	-3.504	-3.993	-2.002	-2.261	-2.116	-2.483	-1.896	-2.231
18(GLE)	-2.157	-2.463	-6.150	-6.844	-3.669	-4.204	-3.568	-3.969	-6.765	-7.463	-3.179	-3.658	-2.062	-2.326	-2.348	-2.720	-1.891	-2.243
19(DB)	-1.440	-1.653	-6.394	-7.084	-3.124	-3.636	-2.517	-2.805	-4.142	-4.629	-2.552	-2.889	-1.857	-2.069	-1.995	-2.305	-2.153	-2.520
20(UCG)	-1.762	-2.019	-6.125	-6.780	-4.290	-4.916	-3.053	-3.384	-6.665	-7.352	-3.342	-3.820	-2.234	-2.518	-2.758	-3.214	-2.271	-2.672
21(ING)	-1.651	-1.889	-8.785	-9.703	-5.871	-6.668	-3.244	-3.630	-5.388	-6.028	-3.020	-3.421	-1.982	-2.235	-2.026	-2.372	-2.090	-2.471
22(SAN)	-1.425	-1.647	-4.571	-5.122	-2.615	-3.021	-3.085	-3.419	-3.043	-3.411	-2.652	-3.002	-1.743	-1.954	-1.623	-1.908	-2.133	-2.480
23(NDA)	-1.373	-1.587	-4.140	-4.640	-3.629	-4.174	-2.062	-2.311	-2.864	-3.203	-1.750	-1.976	-1.315	-1.483	-1.339	-1.568	-1.645	-1.935
24(CS)	-1.544	-1.768	-6.889	-7.688	-3.497	-4.045	-2.196	-2.434	-3.511	-3.896	-2.202	-2.478	-1.574	-1.765	-1.533	-1.783	-1.690	-1.978
25(UBS)	-1.850	-2.107	-7.817	-8.665	-4.891	-5.525	-2.071	-2.313	-3.476	-3.883	-1.792	-2.033	-1.386	-1.553	-1.215	-1.417	-1.744	-2.061
26(MTU)	-2.576	-2.934	-5.676	-6.357	-2.866	-3.216	-1.666	-1.862	-2.374	-2.655	-1.955	-2.221	-1.805	-2.045	-1.795	-2.096	-1.919	-2.270
27(MFG)	-2.225	-2.523	-6.749	-7.545	-3.525	-3.919	-1.863	-2.079	-2.185	-2.434	-1.800	-2.048	-1.546	-1.747	-1.260	-1.465	-1.416	-1.678
28(SMFG)	-2.363	-2.689	-6.736	-7.510	-3.626	-4.043	-1.629	-1.826	-2.190	-2.449	-1.713	-1.948	-1.777	-2.025	-1.816	-2.105	-1.726	-2.036
Average	-1.887	-2.165	-6.567	-7.319	-4.361	-4.959	-2.430	-2.715	-3.784	-4.211	-2.175	-2.470	-1.567	-1.761	-1.579	-1.845	-1.829	-2.144
System	-1.892	-2.128	-5.963	-6.678	-4.053	-4.621	-2.276	-2.607	-3.294	-3.674	-1.944	-2.206	-1.444	-1.655	-1.419	-1.675	-1.778	-2.098

Table 4: Expected Shortfall at 95% and 99% quantiles for each SIFI

Index (SIFI)	2007		2008		2009		2010		2011		2012		2013		2014		2015	
	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%
1(JPM)	-1.585	-1.600	-5.865	-7.386	-4.635	-5.754	-2.048	-2.613	-2.744	-3.420	-1.692	-2.068	-1.149	-1.392	-1.198	-1.439	-1.684	-2.030
2(BAC)	-1.280	-1.292	-6.453	-8.126	-6.655	-8.261	-2.283	-2.912	-3.820	-4.760	-2.828	-3.457	-1.241	-1.503	-1.536	-1.844	-1.860	-2.242
3(BKM)	-1.772	-1.789	-5.674	-7.146	-3.748	-4.653	-1.516	-1.934	-2.620	-3.265	-1.769	-2.163	-1.136	-1.376	-1.243	-1.491	-1.602	-1.931
4(CITI)	-1.647	-1.662	-7.825	-9.855	-7.109	-8.824	-2.515	-3.209	-3.292	-4.102	-2.514	-3.074	-1.444	-1.748	-1.417	-1.701	-1.643	-1.980
5(GS)	-2.324	-2.346	-4.752	-5.985	-3.194	-3.965	-1.652	-2.108	-2.439	-3.039	-1.834	-2.242	-1.312	-1.589	-1.453	-1.745	-1.654	-1.994
6(MS)	-2.401	-2.424	-7.474	-9.413	-4.056	-5.035	-2.332	-2.975	-3.745	-4.666	-2.890	-3.533	-1.580	-1.914	-1.846	-2.216	-1.815	-2.188
7(SST)	-1.902	-1.920	-5.509	-6.938	-5.324	-6.609	-2.084	-2.659	-2.339	-2.915	-1.356	-1.658	-1.026	-1.243	-1.448	-1.738	-1.667	-2.010
8(WFC)	-1.529	-1.544	-4.950	-6.234	-5.829	-7.235	-2.141	-2.731	-2.566	-3.198	-1.381	-1.688	-0.962	-1.165	-1.151	-1.381	-1.411	-1.701
9(RBS)	-2.479	-2.502	-5.305	-6.681	-8.477	-10.522	-2.873	-3.666	-3.810	-4.747	-3.101	-3.791	-2.110	-2.555	-1.810	-2.172	-1.836	-2.213
10(BAR)	-2.689	-2.715	-5.411	-6.814	-6.060	-7.523	-3.141	-4.007	-3.905	-4.866	-2.529	-3.092	-1.552	-1.879	-1.593	-1.912	-1.837	-2.214
11(HSBC)	-1.301	-1.314	-2.683	-3.379	-3.210	-3.985	-1.252	-1.598	-1.917	-2.388	-1.226	-1.499	-1.089	-1.319	-1.280	-1.537	-1.517	-1.828
12(STAN)	-2.373	-2.396	-4.920	-6.195	-3.357	-4.167	-2.067	-2.637	-2.132	-2.657	-2.010	-2.457	-1.377	-1.668	-1.320	-1.584	-2.353	-2.837
13(BOC)	-2.169	-2.190	-3.763	-4.740	-2.390	-2.966	-1.758	-2.242	-2.002	-2.495	-1.460	-1.785	-1.261	-1.527	-1.342	-1.611	-1.939	-2.338
14(ICBC)	-2.272	-2.293	-4.052	-5.103	-2.441	-3.030	-1.700	-2.169	-2.546	-3.172	-1.681	-2.055	-1.187	-1.437	-1.270	-1.525	-1.694	-2.043
15(OCB)	-2.941	-2.970	-4.765	-6.001	-2.647	-3.286	-1.692	-2.159	-2.168	-2.702	-1.755	-2.145	-1.177	-1.425	-1.405	-1.686	-1.712	-2.064
16(BNP)	-1.822	-1.839	-3.990	-5.024	-3.724	-4.623	-2.577	-3.288	-4.421	-5.509	-2.792	-3.413	-1.698	-2.056	-1.906	-2.288	-2.140	-2.580
17(ACA)	-1.854	-1.872	-4.863	-6.124	-3.390	-4.208	-2.891	-3.688	-4.140	-5.159	-3.798	-4.644	-2.017	-2.443	-2.222	-2.667	-1.980	-2.387
18(GLE)	-2.159	-2.180	-4.653	-5.860	-3.723	-4.621	-2.994	-3.820	-4.941	-6.157	-3.723	-4.551	-2.054	-2.488	-2.251	-2.702	-2.076	-2.503
19(DB)	-1.507	-1.521	-4.627	-5.827	-3.567	-4.428	-2.141	-2.731	-3.443	-4.290	-2.613	-3.195	-1.651	-2.000	-1.876	-2.252	-2.162	-2.607
20(UCG)	-1.813	-1.831	-4.388	-5.526	-4.355	-5.406	-2.467	-3.148	-4.864	-6.061	-3.713	-4.539	-2.210	-2.676	-2.761	-3.314	-2.364	-2.850
21(ING)	-1.677	-1.693	-6.157	-7.754	-5.544	-6.882	-2.874	-3.668	-4.532	-5.647	-3.114	-3.806	-1.972	-2.388	-2.096	-2.516	-2.245	-2.706
22(SAN)	-1.565	-1.580	-3.692	-4.650	-2.825	-3.507	-2.493	-3.181	-2.606	-3.247	-2.719	-3.324	-1.639	-1.984	-1.724	-2.070	-2.047	-2.468
23(NDA)	-1.510	-1.524	-3.348	-4.217	-3.788	-4.702	-1.859	-2.373	-2.396	-2.985	-1.753	-2.142	-1.305	-1.581	-1.389	-1.668	-1.710	-2.062
24(CS)	-1.577	-1.592	-5.358	-6.747	-3.815	-4.736	-1.775	-2.265	-2.729	-3.400	-2.146	-2.624	-1.482	-1.795	-1.513	-1.816	-1.697	-2.046
25(UBS)	-1.811	-1.828	-5.685	-7.159	-4.417	-5.482	-1.809	-2.308	-2.878	-3.586	-1.869	-2.285	-1.295	-1.568	-1.225	-1.470	-1.866	-2.250
26(MTU)	-2.520	-2.544	-4.571	-5.756	-2.436	-3.023	-1.467	-1.872	-1.989	-2.478	-2.064	-2.523	-1.863	-2.257	-1.821	-2.186	-2.069	-2.494
27(MFG)	-2.101	-2.121	-5.338	-6.722	-2.744	-3.406	-1.606	-2.049	-1.764	-2.198	-1.929	-2.358	-1.563	-1.893	-1.242	-1.490	-1.541	-1.858
28(SMFG)	-2.295	-2.317	-5.187	-6.532	-2.905	-3.606	-1.469	-1.875	-1.835	-2.287	-1.824	-2.230	-1.928	-2.335	-1.754	-2.105	-1.826	-2.201
Average	-1.960	-1.979	-5.045	-6.353	-4.156	-5.159	-2.124	-2.710	-3.021	-3.764	-2.289	-2.798	-1.510	-1.829	-1.610	-1.933	-1.855	-2.237

Table 5: Exposure  $\Delta\text{CoVaR}$  at 95% and 99% quantiles for each SIFI

Index (SIFI)	2007		2008		2009		2010		2011		2012		2013		2014		2015	
	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%
1(JPM)	-1.532	-1.725	-6.620	-7.567	-4.509	-5.104	-2.304	-2.640	-3.250	-3.638	-1.764	-1.917	-1.214	-1.352	-1.199	-1.355	-1.677	-1.903
2(BAC)	-1.299	-1.455	-8.815	-9.857	-7.209	-8.064	-2.701	-3.075	-5.326	-5.865	-2.610	-2.867	-1.306	-1.455	-1.530	-1.731	-1.917	-2.167
3(BKM)	-1.584	-1.800	-6.893	-7.809	-4.122	-4.604	-1.669	-1.917	-3.237	-3.607	-1.851	-2.012	-1.190	-1.327	-1.209	-1.372	-1.616	-1.831
4(CITI)	-1.851	-2.051	-11.653	-12.917	-9.860	-10.773	-2.561	-2.973	-4.269	-4.734	-2.554	-2.783	-1.548	-1.721	-1.447	-1.632	-1.678	-1.899
5(GS)	-2.144	-2.427	-6.211	-6.978	-2.825	-3.235	-1.851	-2.121	-3.167	-3.511	-1.868	-2.035	-1.370	-1.527	-1.445	-1.635	-1.718	-1.940
6(MS)	-2.425	-2.716	-10.392	-11.599	-3.908	-4.429	-2.713	-3.095	-4.847	-5.376	-3.059	-3.322	-1.550	-1.740	-1.771	-2.012	-1.943	-2.188
7(SST)	-1.695	-1.926	-7.093	-7.983	-5.950	-6.634	-2.297	-2.638	-2.677	-3.008	-1.455	-1.578	-1.030	-1.154	-1.455	-1.644	-1.775	-1.999
8(WFC)	-1.509	-1.695	-5.081	-5.880	-6.432	-7.180	-2.323	-2.674	-2.928	-3.290	-1.467	-1.592	-1.011	-1.126	-1.113	-1.264	-1.436	-1.626
9(RBS)	-2.656	-2.957	-8.759	-9.616	-11.107	-12.195	-3.084	-3.555	-5.008	-5.546	-2.989	-3.271	-2.410	-2.664	-1.792	-2.028	-1.986	-2.233
10(BAR)	-2.844	-3.171	-7.487	-8.361	-5.857	-6.635	-3.689	-4.204	-4.838	-5.390	-2.509	-2.738	-1.761	-1.948	-1.690	-1.898	-1.914	-2.161
11(HSBC)	-1.261	-1.419	-3.012	-3.445	-3.330	-3.742	-1.440	-1.645	-2.302	-2.573	-1.258	-1.369	-1.244	-1.375	-1.340	-1.507	-1.605	-1.809
12(STAN)	-2.066	-2.354	-6.096	-6.891	-3.057	-3.489	-2.224	-2.563	-2.482	-2.783	-2.157	-2.339	-1.655	-1.820	-1.458	-1.631	-2.731	-3.048
13(BOC)	-2.102	-2.366	-4.618	-5.225	-2.047	-2.354	-1.948	-2.236	-2.433	-2.716	-1.549	-1.681	-1.439	-1.591	-1.279	-1.454	-2.126	-2.387
14(ICBC)	-2.026	-2.302	-4.634	-5.289	-2.264	-2.577	-1.926	-2.204	-2.991	-3.351	-1.799	-1.952	-1.387	-1.530	-1.251	-1.417	-1.835	-2.063
15(CCB)	-2.498	-2.855	-5.665	-6.434	-2.469	-2.809	-1.823	-2.100	-2.563	-2.869	-1.884	-2.044	-1.381	-1.522	-1.382	-1.565	-1.849	-2.079
16(BNP)	-1.777	-1.998	-5.258	-5.902	-3.526	-4.004	-3.045	-3.467	-5.516	-6.140	-2.740	-2.993	-1.814	-2.018	-2.029	-2.278	-2.103	-2.391
17(ACA)	-1.914	-2.139	-6.527	-7.313	-3.376	-3.811	-3.533	-4.007	-5.586	-6.171	-3.743	-4.088	-2.069	-2.312	-2.213	-2.504	-1.984	-2.250
18(GLE)	-2.202	-2.464	-6.202	-6.953	-3.734	-4.212	-3.570	-4.061	-6.772	-7.470	-3.413	-3.751	-2.131	-2.378	-2.446	-2.740	-1.983	-2.263
19(DB)	-1.471	-1.654	-6.445	-7.192	-3.186	-3.644	-2.519	-2.870	-4.147	-4.633	-2.717	-2.954	-1.912	-2.110	-2.077	-2.322	-2.249	-2.540
20(UCG)	-1.800	-2.020	-6.174	-6.882	-4.366	-4.926	-3.055	-3.459	-6.671	-7.358	-3.576	-3.913	-2.308	-2.573	-2.878	-3.239	-2.377	-2.695
21(ING)	-1.686	-1.890	-8.853	-9.847	-5.968	-6.680	-3.247	-3.718	-5.394	-6.034	-3.216	-3.499	-2.048	-2.285	-2.118	-2.391	-2.190	-2.492
22(SAN)	-1.458	-1.648	-4.612	-5.208	-2.664	-3.027	-3.087	-3.496	-3.046	-3.414	-2.824	-3.071	-1.798	-1.995	-1.698	-1.923	-2.224	-2.499
23(NDA)	-1.405	-1.588	-4.177	-4.718	-3.695	-4.182	-2.063	-2.368	-2.868	-3.206	-1.861	-2.020	-1.359	-1.516	-1.400	-1.581	-1.721	-1.951
24(CS)	-1.577	-1.768	-6.948	-7.814	-3.563	-4.054	-2.197	-2.488	-3.514	-3.900	-2.337	-2.532	-1.624	-1.802	-1.599	-1.797	-1.766	-1.994
25(UBS)	-1.887	-2.107	-7.880	-8.798	-4.968	-5.535	-2.072	-2.369	-3.480	-3.886	-1.910	-2.080	-1.429	-1.585	-1.269	-1.429	-1.827	-2.078
26(MTU)	-2.629	-2.935	-5.726	-6.464	-2.909	-3.221	-1.667	-1.907	-2.376	-2.657	-2.086	-2.273	-1.867	-2.091	-1.874	-2.112	-2.011	-2.290
27(MFG)	-2.269	-2.524	-6.808	-7.670	-3.573	-3.925	-1.865	-2.128	-2.187	-2.436	-1.921	-2.097	-1.598	-1.786	-1.314	-1.476	-1.485	-1.692
28(SMFG)	-2.411	-2.690	-6.793	-7.631	-3.677	-4.050	-1.630	-1.871	-2.193	-2.452	-1.828	-1.994	-1.841	-2.073	-1.892	-2.121	-1.807	-2.053
Average	-1.928	-2.166	-6.623	-7.437	-4.434	-4.968	-2.432	-2.780	-3.788	-4.215	-2.319	-2.527	-1.618	-1.799	-1.649	-1.859	-1.912	-2.161

Table 6: Marginal Expected Shortfall at 95% and 99% quantiles for each SIFI

## References

- Acharya, V. V., Pedersen, L. H., Philippon, T., and Richardson, M. (2017). Measuring systemic risk. *The Review of Financial Studies*, 30(1):2–47.
- Adrian, T. and Brunnermeier, M. (2008). Covar. *Federal Reserve Bank of New York Staff Report No. 348*.
- Azzalini, A. and Capitanio, A. (2003). Distributions generated by perturbation of symmetry with emphasis on a multivariate skew t-distribution. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 65(2):367–389.
- Benoit, S., Colletaz, G., Hurlin, C., and Perignon, C. (2013). A theoretical and empirical comparison of systemic risk measures. Technical report, FIN-2014-1030.
- Bernardi, M., Maruotti, A., and Petrella, L. (2013). Multivariate markov-switching models and tail risk interdependence. *Working Paper*.
- Bluhm, C. and Wagner, C. (2011). Valuation and risk management of collateralized debt obligations and related securities. *Annual Reviews*.
- Borak, S., Misiorek, A., and Weron, R. (2011). Models for heavy-tailed asset returns. *Statistical Tools for Finance and Insurance. 2nd Ed. Berlin: Springer*, pages 21–55.
- Brownlees, C. and Engle, R. F. (2016). Srisk: A conditional capital shortfall measure of systemic risk. *The Review of Financial Studies*, 30(1):48–79.
- Cao, Z. (2013). Multi-covar and shapley value: a systemic risk measure. *Banque de France Working Paper Series*.
- Chan, J. C. and Kroese, D. P. (2010). Efficient estimation of large portfolio loss probabilities in t-copula models. *European Journal of Operational Research*, 205(2):361–367.
- Chen, C. Y.-H., Härdle, W. K., and Okhrin, Y. (2017). Tail event driven networks of SIFIs. SFB 649 Discussion Paper 2017-004, Sonderforschungsbereich 649, Humboldt Universität zu Berlin, Germany. available at <http://sfb649.wiwi.hu-berlin.de/papers/pdf/SFB649DP2017-004.pdf>.
- Chen, C. Y.-H. and Nasekin, S. (2017). The systemic risk of central SIFIs. SFB 649 Discussion Paper 2017-021, Sonderforschungsbereich 649, Humboldt Universität zu Berlin, Germany. available at <http://sfb649.wiwi.hu-berlin.de/papers/pdf/SFB649DP2017-021.pdf>.

- Choroś, B., Ibragimov, R., and Permiakova, E. (2010). Copula estimation. *Copula theory and its applications*, pages 77–91.
- Choroś-Tomczyka, B., Härdle, W. K., and Overbeck, L. (2012). Copula Dynamics in CDOs. SFB 649 Discussion Paper 2012-032, Sonderforschungsbereich 649, Humboldt Universität zu Berlin, Germany. available at <http://sfb649.wiwi.hu-berlin.de/papers/pdf/SFB649DP2012-032.pdf>.
- Christoffersen, P., Jacobs, K., Jin, X., and Langlois, H. (2013). Dynamic dependence in corporate credit. *Dynamic Dependence in Corporate Credit,” Rotman School of Management Working Paper*, (2314027).
- Claessens, S. and Forbes, K. (2001). International financial contagion: An overview of the issues and the book. In *International financial contagion*, pages 3–17. Springer.
- Creal, D. D. and Tsay, R. S. (2015). High dimensional dynamic stochastic copula models. *Journal of Econometrics*, 189(2):335–345.
- De Luca, G. and Riveccio, G. (2012). Multivariate tail dependence coefficients for archimedean copulae. In *Advanced Statistical Methods for the Analysis of Large Data-Sets*, pages 287–296. Springer.
- Dittmar, R. (2010). Cross-market and cross-firm effects in implied default probabilities and recovery values. In *Financial Economics in Rio 2010*.
- Embrechts, P., Frey, R., and McNeil, A. (2005). Quantitative risk management. *Princeton Series in Finance, Princeton*, 10:4.
- Embrechts, P., Lindskog, F., and McNeil, A. (2001). Modelling dependence with copulas. *Rapport technique, Département de mathématiques, Institut Fédéral de Technologie de Zurich, Zurich*.
- Engle, R. F. and Manganelli, S. (2004). Caviar: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics*, 22(4):367–381.
- Fantazzini, D. (2008). Dynamic copula modelling for value at risk. *Frontiers in Finance and Economics*, 5(2):72–108.
- Franke, J., Härdle, W. K., and Hafner, C. M. (2004). *Statistics of financial markets*, volume 2. Springer.

- Frey, R. and McNeil, A. J. (2001). Modelling dependent defaults. Presented at the Conference on “Statistical and Computational Problems in Risk Management”, University of Rome “La Sapienza”.
- Genest, C., Ghoudi, K., and Rivest, L.-P. (1995). A semiparametric estimation procedure of dependence parameters in multivariate families of distributions. *Biometrika*, 82(3):543–552.
- Girardi, G. and Ergün, A. T. (2013). Systemic risk measurement: Multivariate garch estimation of covar. *Journal of Banking & Finance*, 37(8):3169–3180.
- Granger, C. W., Teräsvirta, T., and Patton, A. J. (2006). Common factors in conditional distributions for bivariate time series. *Journal of Econometrics*, 132(1):43–57.
- Hansen, B. E. (1994). Autoregressive conditional density estimation. *International Economic Review*, pages 705–730.
- Hartmann, P., Straetmans, S., and De Vries, C. G. (2004). Asset market linkages in crisis periods. *The Review of Economics and Statistics*, 86(1):313–326.
- He, X. and Gong, P. (2009). Measuring the coupled risks: A copula-based cvar model. *Journal of Computational and Applied Mathematics*, 223(2):1066–1080.
- Hull, J. C. (2006). *Options, futures, and other derivatives*. Pearson Education India.
- Idier, J., Lamé, G., and Mésonnier, J.-S. (2014). How useful is the marginal expected shortfall for the measurement of systemic exposure? a practical assessment. *Journal of Banking & Finance*, 47:134–146.
- Joe, H. (1990). Multivariate concordance. *Journal of multivariate analysis*, 35(1):12–30.
- Joe, H. (1997). *Multivariate models and multivariate dependence concepts*. CRC Press.
- Joe, H. (2014). *Dependence modeling with copulas*. CRC Press.
- Judd, K. L. (1998). *Numerical methods in economics*. MIT press.
- Kahaner, D., Moler, C., and Nash, S. (1989). Numerical methods and software. *Englewood Cliffs: Prentice Hall*.
- Kalemanova, A., Schmid, B., and Werner, R. (2007). The normal inverse gaussian distribution for synthetic cdo pricing. *The journal of derivatives*, 14(3):80–94.



- Kim, G., Silvapulle, M. J., and Silvapulle, P. (2007). Comparison of semiparametric and parametric methods for estimating copulas. *Computational Statistics & Data Analysis*, 51(6):2836–2850.
- Kolman, M. (2014). A one-factor copula-based model for credit portfolios. *Journal of Risk*, 17(2).
- Krupskii, P. and Joe, H. (2013). Factor copula models for multivariate data. *Journal of Multivariate Analysis*, 120:85–101.
- Lin, T.-I., Wu, P. H., McLachlan, G. J., and Lee, S. X. (2013). The skew-t factor analysis model. *arXiv preprint arXiv:1310.5336*.
- Löffler, G. and Raupach, P. (2016). Pitfalls in the use of systemic risk measures. *Journal of Financial and Quantitative Analysis (JFQA)*. Forthcoming.
- Lu, M.-J., Chen, C. Y.-H., and Härdle, W. K. (2015). Copula-based factor model for credit risk analysis. *Review of Quantitative Finance and Accounting*, pages 1–23.
- Mainik, G. and Schaanning, E. (2014). On dependence consistency of covar and some other systemic risk measures. *Statistics & Risk Modeling*, 31(1):49–77.
- McNeil, A. J., Frey, R., and Embrechts, P. (2015). *Quantitative risk management: Concepts, techniques and tools*. Princeton university press.
- Oh, D. H. and Patton, A. J. (2013). Simulated method of moments estimation for copula-based multivariate models. *Journal of the American Statistical Association*, 108(502):689–700.
- Oh, D. H. and Patton, A. J. (2016). High-dimensional copula-based distributions with mixed frequency data. *Journal of Econometrics*, 193(2):349–366.
- Oh, D. H. and Patton, A. J. (2017a). Modeling dependence in high dimensions with factor copulas. *Journal of Business & Economic Statistics*, 35(1):139–154.
- Oh, D. H. and Patton, A. J. (2017b). Time-varying systemic risk: Evidence from a dynamic copula model of cds spreads. *Journal of Business & Economic Statistics*, pages 1–15.
- Pagano, M. S. and Sedunov, J. (2014). What is the relation between systemic risk exposure and sovereign debt. *Social Science Research Network*, 2298273:1–43.

- Patton, A. J. et al. (2012). Copula methods for forecasting multivariate time series. *Handbook of economic forecasting*, 2:899–960.
- Reboredo, J. C. and Ugolini, A. (2015). Systemic risk in european sovereign debt markets: A covar-copula approach. *Journal of International Money and Finance*, 51:214–244.
- Schmidt, R. and Stadtmüller, U. (2006). Non-parametric estimation of tail dependence. *Scandinavian Journal of Statistics*, 33(2):307–335.
- Shamiri, A., Hamzah, N., and Pirmoradian, A. (2011). Tail dependence estimate in financial market risk management: clayton-gumbel copula approach. *Sains Malaysiana*, 40(8):927–935.
- Sklar, M. (1959). Fonctions de repartition an dimensions et leurs marges. *Publ. Inst. Statist. Univ. Paris*, 8:229–231.
- Vrins, F. D. (2009). Double t copula pricing of structured credit products: Practical aspects of a trustworthy implementation. *Journal of Credit Risk*.
- Yang, R., Qin, X., and Chen, T. (2009). Cdo pricing using single factor mg-nig copula model with stochastic correlation and random factor loading. *Journal of Mathematical Analysis and Applications*, 350(1):73–80.

# Declaration of Academic Honesty

I do solemnly declare, that I Florian Reichert, towards the Ladislaus von Bortkiewicz Chair of Statistics of the Humboldt-University of Berlin, prepared this thesis titled

*Measuring systemic risk in financial institutions: A factor-copula framework*

independently and that the thoughts taken directly or indirectly from other sources are indicated accordingly. The work has not been submitted to any other examination authority and also not yet been published.

Berlin, 4 September 2017